Forces and Newton's laws of motion

Unit 3

Contents

Forces are all around us. From keeping us standing on the Earth, to the Earth moving around the Sun. We experience forces every day of our lives.

This unit looks at different types of forces, how they interact and what effect they have on motion. This is a large topic encompassing some of the most important work ever carried out by Physicists. You will look into Newton's laws, Hooke's work on springs, and even learn how to calculate your mass and weight on different planets.

3.1 Forces in nature

By the end of this section you should be able to:

- List some of the forces that occur in nature and categorise them as contact or non-contact.
- State Newton's first law.
- Explain the relationship between mass and inertia.
- State Hooke's law and distinguish between elastic and inelastic materials.
- Experimentally determine and describe the force constant of a spring.

Figure 3.1 *Weight is a common force we experience every day.*

What are forces?

In simple terms, a force is a **push** or a **pull**. You might push a book across the desk or gravity might pull objects towards the centre of the Earth.

There are plenty of different examples of forces. However, if you look deeper, forces fall into just four groups:

- Electromagnetic forces, dealing with charged objects, atomic interactions and whenever objects come into contact.
- Gravity, which relates to all objects that have mass, from an apple falling to the ground to the Earth orbiting the Sun.
- Finally, two forces dealing with interactions within the nucleus of atoms. These are called the strong nuclear force and the weak nuclear force. Although very important we rarely encounter these forces in our day to day lives.

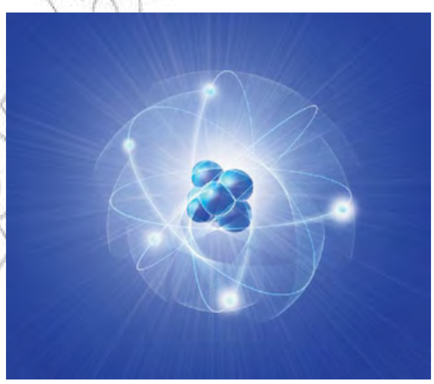


Figure 3.2 *Forces play an important role in keeping atoms together.*

Below are some examples of common forces.

Table 3.1 *Some examples of forces*

Friction	Drag forces (including air resistance and water resistance)
Electrostatic attraction or repulsion	Thrust
Buoyant force (upthrust)	Gravitational attraction
Weight	Tension
Contact force (reaction)	Magnetic attraction or repulsion

Activity 3.1: Categorising forces

Categorise all the forces listed in Table 3.1 as contact or non-contact.

All forces are vector quantities. This means they all have both a **magnitude** and a **direction,** and are often represented in diagrams as arrows. The size of the arrow represents the magnitude of the force and the way it is pointing shows the direction it is acting. The SI derived unit of force is the newton (N).

Figure 3.3 *Forces pull stars together to form gigantic galaxies.*

Figure 3.4 on the next page, is called a **free body diagram**. These kinds of diagrams are really useful when dealing with forces. It

DID YOU KNOW?

All forces are measured in **newtons**, named after Sir Isaac Newton (more on him later). He was born in 1642 and in his famous book *Principia Mathematica* he made significant advances in understanding motion. He also developed key theories on gravity and optics, and invented an entire new branch of mathematics: calculus.

KEY WORDS

pull *movement towards a force*

push *movement away from a force*

contact forces *forces where objects must touch before the force has an effect*

newton *SI unit of force*

non-contact forces *forces where objects are not required to touch for the force to have an effect*

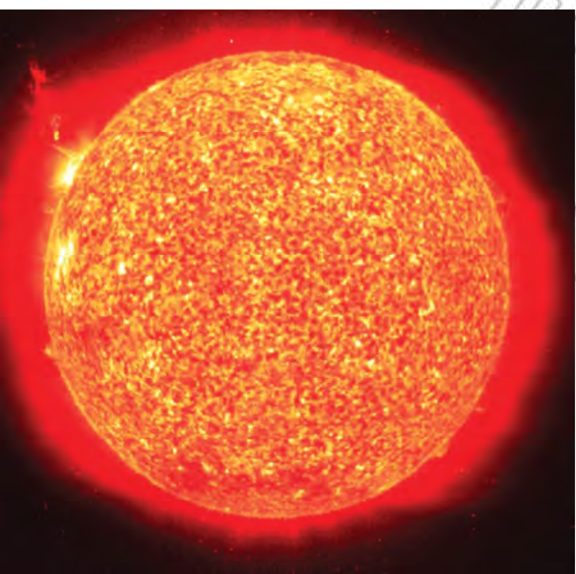


Figure 3.6 *Despite being 150 million km away the Sun's gravity still has a significant influence on the motion of the Earth.*

is important that you consider all the forces acting and draw the arrows approximately to scale. In this case the weight of the stone is greater than the air resistance.

Contact or non-contact

Forces can be categorised as either **contact** or **non-contact**. Some forces act over a distance and so the objects involved do not need to be touching. Other forces need objects to touch before their effects can be noticed.

If you push your hands together you can feel a contact force (this is really an electrostatic repulsion between the electrons in the atoms in your hands). The same is true when you kick a ball.

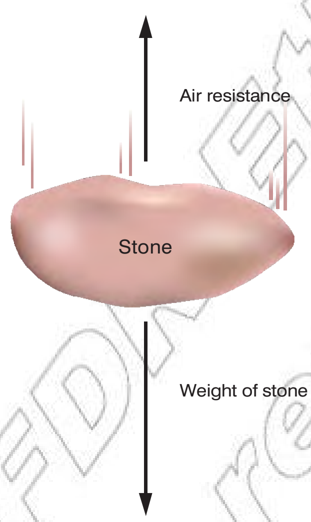


Figure 3.4 *Forces acting on a stone falling through the air*

Figure 3.5 *Kicking a ball demonstrates a contact force.*

Several forces act over a distance, the most obvious being gravitational attraction. The Earth is kept in orbit around the Sun even though they are 150 million km apart!

It is not just gravity; magnetic forces can also act over distances, for example, two magnets attracting each other.

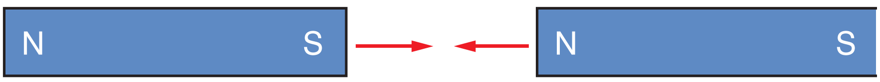


Figure 3.7 *Magnets can attract or repel each other without being in contact.*

What effect do forces have?

The famous ancient Greek, Aristotle, did a great deal to help develop the idea of science. However, he got forces all wrong! He thought that forces were needed to make objects move, that is, there cannot be any movement unless a force is acting.

This idea makes a lot of sense in our experience. If we push a block along it will keep on moving, but if we stop pushing the block it will slow down and stop. The problem is that on Earth whenever objects are in motion there are other forces acting, namely weight, friction and/or drag. These forces have an effect on the motion of the object.

It is true that forces and motion are linked but forces *do not simply make objects move.*

It was not until the famous English physicist, Sir Isaac Newton, came along, some 2000 years after Aristotle, that we developed a more complete understanding of forces. Newton took some of the ideas developed by Galileo and constructed three laws that describe how motion and forces are related.

Newton's first law of motion explains what effect forces have on objects. It states:

- **An object will remain at rest or travelling at a constant velocity unless acted upon by an external force.**

This takes a bit of reading but what it means is that forces don't make objects move but they do make objects change the way they are moving.

An object will remain at rest unless a force makes it start to move. It will then continue to move at the same velocity until another force slows it down. So using our block example from earlier, when we stop pushing it the block slows down because friction is acting on it. If there was no friction it would continue at the same velocity until another force acted on it.

The use of the term velocity here is also important. It means an object moving around a curve or in a circle must have a force acting on it. Whenever an object moves in a circle its velocity is changing (because velocity is a vector quantity) and so according to Newton's first law there must be a force acting on it.

Newton's first law means a force is always required to make an object:

- **speed up**
- **slow down**
- **or change direction.**

If an object is not doing any of these, then we can conclude there are no overall forces acting on it. This might mean remaining stationary but it also means travelling at a steady speed in a straight line.

Mass and inertia

Newton's first law means that objects have a tendency to resist any changes to their motion. They will remain stationary or at constant velocity unless a force acts on them.

This is referred to as the **inertia** of an object. It is defined as:

- **The property of an object to remain at rest or moving at a steady speed in a straight line.**

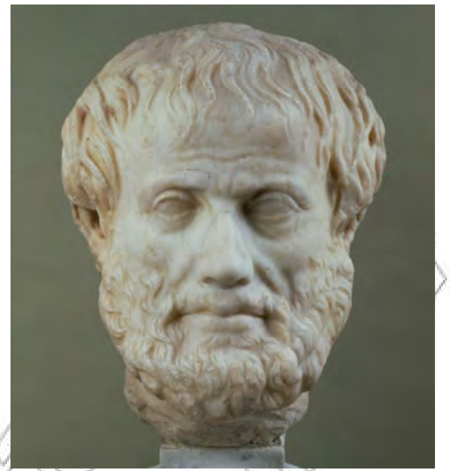


Figure 3.8 *Aristotle developed some excellent theories but his ideas about forces were wrong!*

Figure 3.9 *Sir Isaac Newton – perhaps the greatest physicist of all time.*

Think about this...

Newton's first law means if you were to throw a tennis ball in space, far away from any stars and planets, it would continue to travel at a steady speed in a straight line forever! (Well until it got near another object and then its gravity would start to have an effect).

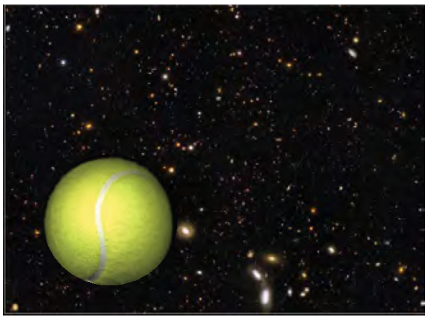


Figure 3.10 *With no friction or air resistance to slow it down, a ball thrown in space will travel at a steady speed in a straight line.*

You may have experienced this on a bus or train. If you are standing still and the vehicle moves you tend to fall backwards. This is because as it moves your feet are pulled along due to friction, but the rest of your body resists this change in motion; it wants to stay at rest.

The same is true when the bus/train stops suddenly; you tend to 'fly forward'. You're not really flying forward, you just keep moving at the same speed as the vehicle slows down.

The inertia of an object depends on its mass. **The greater the mass of the object, the greater its inertia.**

This is why it is easy to kick a small stone. Because it has a small mass and so a small inertia, only a small force is required to change the motion of the stone. However, a large boulder has a great deal more mass. If you kicked a boulder chances are it wouldn't move (and you'd have a sore toe!). It has much more mass, so it has a much greater inertia and a much larger force is required to change its motion.

KEY WORDS

inertia *the tendency of an object to resist changes to its motion*

Figure 3.11 *The greater the mass the greater the inertia. The large boulder has a much greater inertia.*

Activity 3.2: Observing inertia

Try these simple observations (Figure 3.12).

- Place a book on a cloth on a smooth table. Pull the cloth quickly. The book remains at rest.
- Place a coin on a small card. Support the card on the edges of a table so that its sides stick out. Hit the card firmly with one finger. The coin stays where it is.
- Put some water in a bucket or can. Spin it around quickly, in a vertical circle. Although the can is upside down at the top of the circle, no water falls out.

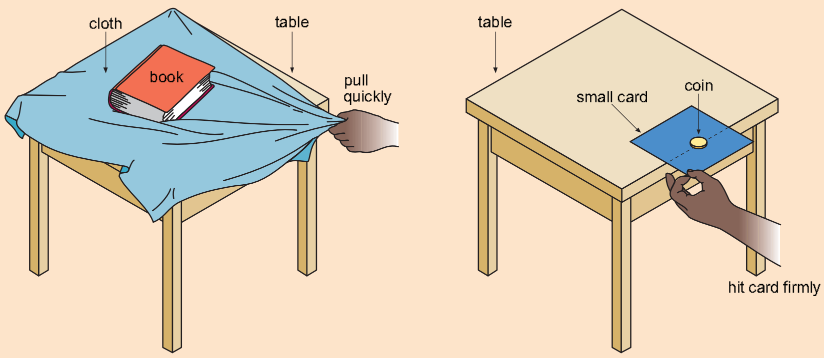


Figure 3.12 *Demonstrating inertia*

DID YOU KNOW?

Inertia comes from the Latin word, "iners", meaning idle, or lazy. Newton used this word to illustrate that objects were lazy; they did not want to move unless a force was applied to them.

Other effects of forces

If more than one force acts on an object it can also *change the shape* of the object. Two parallel equal and opposite forces can either stretch or compress an object.

Forces can also twist or bend an object if applied in different directions.

Robert Hooke was another physicist working in London around the same time as Newton. He was investigating methods for making more precise clocks. He was interested in the effect forces had on springs.

Hooke used springs fixed at one end (this provided an upward force) and applied a force to the bottom of the spring to stretch it (this force is sometimes called the **load**).

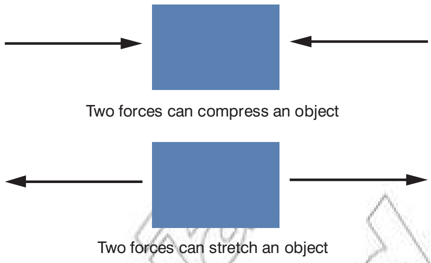


Figure 3.13 *Some possible effects of two equal and opposite forces*

KEY WORDS

load *a force applied to a spring*

extension *the increase in length of a spring*

Figure 3.14 *Robert Hooke was a great scientist and was a rival of Isaac Newton.*

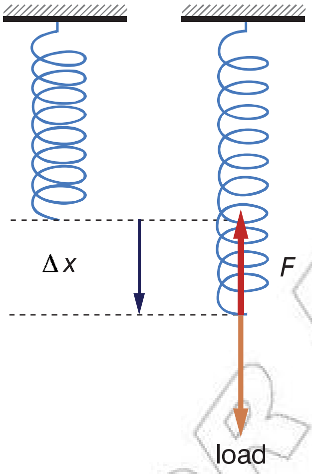


Figure 3.15 *Applying a force to a spring fixed at one end causes it to extend.*

Hooke found that the greater the force applied to the spring the greater the **extension**. Not only that, he found that the extension of the spring was directly proportional to the force applied. This is often referred to as **Hooke's law**.

This means when he applied twice the force the spring would extend twice as far. Three times the force, the spring would extend three times as far.

Hooke's experiments are easy to repeat in a lab. Figure 3.17 on the next page, shows a simple experimental arrangement you could use to test his findings.

Table 3.2 *Some results from an experiment on stretching a spring*

Force applied (N)	Length of spring (cm)	Extension (cm)
0	10.0	0.0
1	11.5	1.5
2	13.0	3.0
3	14.5	4.5
4	16.0	6.0
5	18.5	8.5
6	22.0	12.0

DID YOU KNOW?

Hooke is perhaps best known for Hooke's law (also called the law of elasticity), but like Newton he made several other valuable contributions. He is often described as the father of microscopy, making several important discoveries. Hooke also came up with the term cell to describe the basic unit of life.

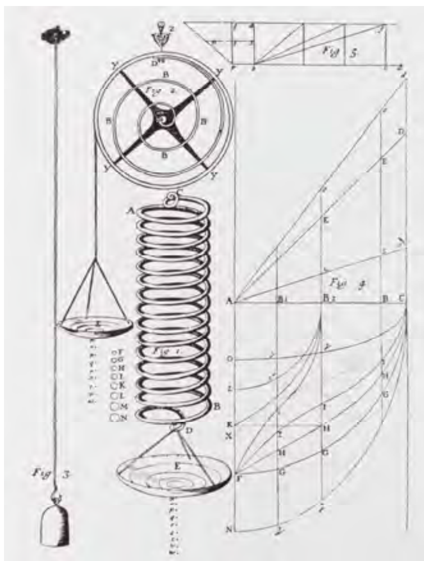


Figure 3.16 *Some of Robert Hooke's original drawings of his experiment*

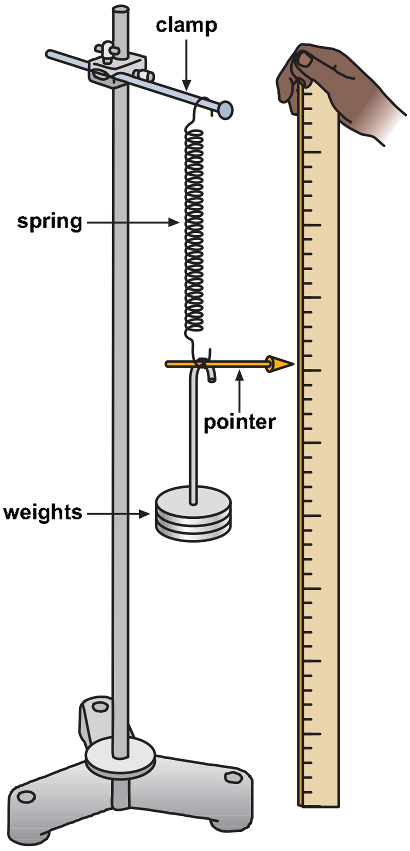


***Figure 3.17** Investigating how force affects the extension of a spring*

Think about this...

Extending twice as far does not mean the spring is now double its length. It just means the extension is twice the size. Take a spring 15 mm long; if 2 N caused a 3 mm extension then 4 N would cause a 6 mm extension. With a load of 4 N the spring would be 21 mm long.

KEY WORDS

Hooke's Law *the force applied to a spring is directly proportional to its extension up to the elastic limit*

directly proportional *a relationship where both variables increase (or decrease) at the same time*

Plotting these results on a graph will produce one like that in Figure 3.18. With Hooke's law experiments it is not uncommon to see it the other way around, with extension plotted against force applied, so make sure you look carefully at the axis!

Any relationship that is **directly proportional** will produce a straight line graph with the line going through the **origin**. However, it is worth remembering it does not have to be at 45°. Figure 3.19 shows three directly proportional relationships.

Looking at Figure 3.19, what is different about the springs to produce different slopes? Some springs are stiffer than others. A stiffer spring will not extend as far when a force is applied to it. Looking at the graph, which is the stiffest spring?

If you answered spring A you'd be correct. Spring C is the least stiff; it is the easiest to extend. Let's look at why, but this time just using two springs instead of three.

Figure 3.20 shows the results collected for two different springs. Spring A is stiffer than spring B.

Consider the same force applied to each spring – force *F*. You can see from the second graph that this force causes spring B to extend more than spring A. Therefore you can conclude that spring A is stiffer than spring B.

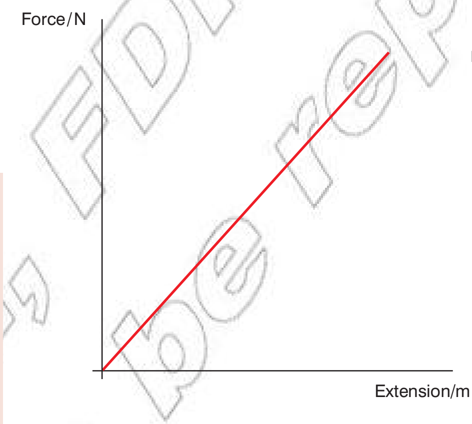


***Figure 3.18** A graph showing that force is directly proportional to extension*

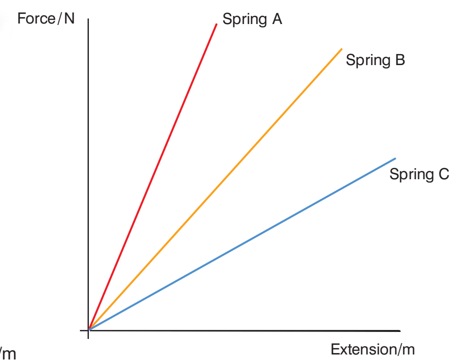


***Figure 3.19** Three directly proportional relationships for three different springs*

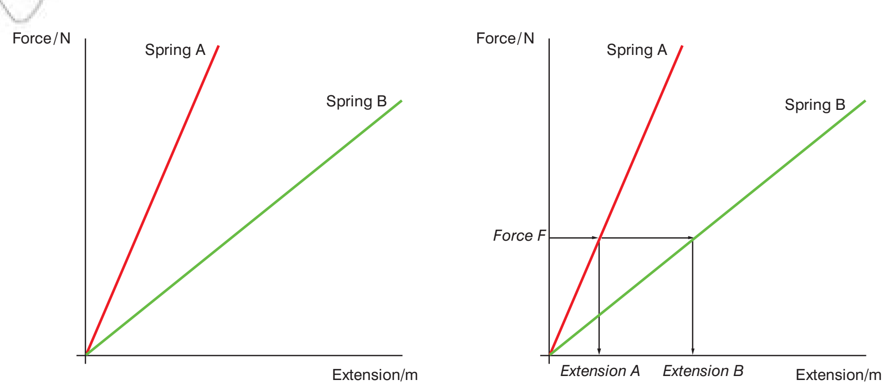


***Figure 3.20** Results collected for two different springs*

The **spring constant** is a measure of the **stiffness** of a spring. It is given the symbol *k*. A stiff spring might have a spring constant of 1000 N/m and a less stiff spring might have a spring constant of 15 N/m.

You can determine the spring constant of any given spring by using the force–extension graph.

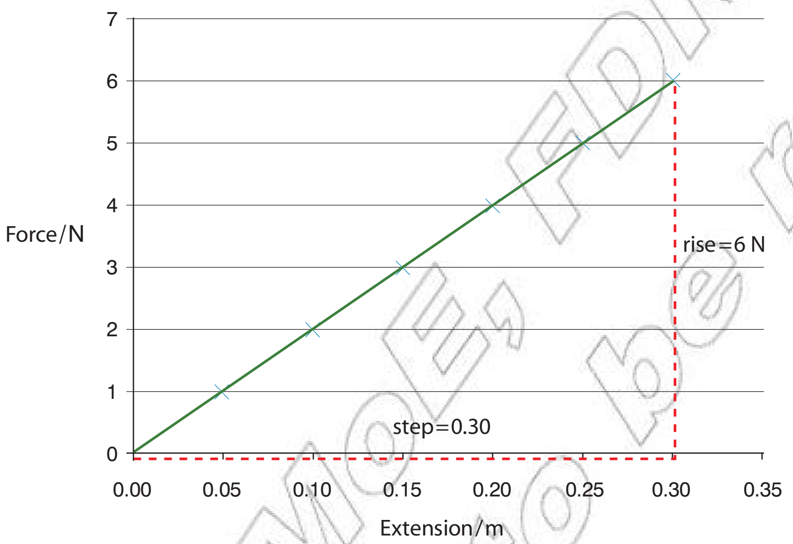
The **gradient** of the line is equal to the spring constant. The steeper the line, the higher the gradient, the greater the spring constant and the stiffer the spring!

Using the data and graph below we can determine the spring constant for the spring.

***Table 3.3** Typical force and extension data*

Force (N)	Extension (m)
0	0.00
1	0.05
2	0.10
3	0.15
4	0.20
5	0.25
6	0.30

***Figure 3.23** Force–extension graph using data from Table 3.3*



The gradient of the line = rise/step *State principle or equation to be used (determine the gradient of the line)*

gradient = 6 N / 0.3 m *Substitute in known values and complete calculation*

gradient = 20 N/m *Clearly state the answer with unit*

Therefore, *k* = 20 N/m. *Make clear the gradient is also equal to k*

Spring balances

The relationship between force and extension is used to great effect in **spring balances**. These are very simple devices designed to measure forces. They are often used to determine the weight of an object.

Think about this...

The spring constant of 15 N/m means you would need to apply a force of 15 N to extend the spring by 1 m. 30 N would cause an extension of 2 m, etc. If *k* = 1000 N/m, then 1000 N would be needed to extend the spring by 1 m. 500 N would cause a 50 cm extension, etc.

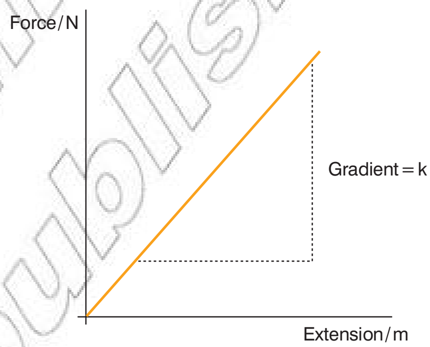


***Figure 3.21** Using a force–extension graph to determine the spring constant*

***Figure 3.22** The springs used in car suspension systems need to have a high spring constant.*

KEY WORDS

origin *the point of intersection of the axes of a graph*

Figure 3.24 *Two different examples of spring balances*

DID YOU KNOW?

Spring balances are often called newtonmeters (or forcemeters). That definitely would not have pleased Hooke! He and Newton were scientific rivals and did not get on at all well.

Think about this...

What would be different about the spring constant of a spring in a spring balance used to weigh heavy objects?

KEY WORDS

gradient *the slope of a line on a graph*

spring balance *device used to measure force via the extension of a spring*

spring constant *a measure of the stiffness of a spring*

stiffness *the amount of force required to stretch or compress a spring*

Spring balances work on the principle that the greater the force applied the greater the extension. This means it is easy to construct a simple scale and pointer next to the spring. When a force is applied (e.g. the weight of an object) the spring will extend to a pre-determined length.

Activity 3.3: Making a spring balance

You can make a spring balance of your own.

- You need a spring, and a container for the objects you are going to weigh (Figure 3.25).
- You also need a scale, next to the spring. Make a cardboard pointer, and attach it to the bottom of the spring, so that it will move past the scale.
- First, you must **calibrate** the spring balance. Hang some known loads on the meter. Mark their values on the scale. Mark the scale in equal divisions.
- Now use your meter to weigh other objects.

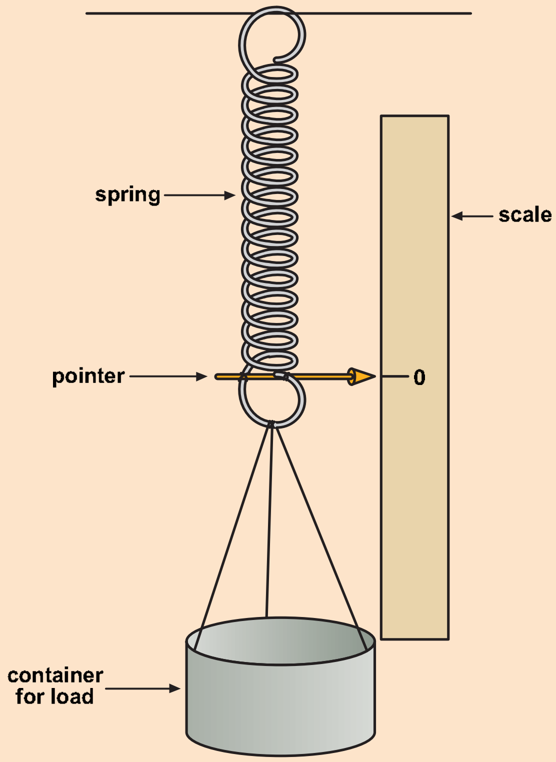


Figure 3.25 *Making a spring balance*

The elastic limit

If we keep on applying force will the spring keep extending forever? Obviously at some point the spring will break, but before it does it behaves slightly differently. It begins to stretch more easily and eventually it will stretch so far that it will not return to its original length when the force is removed.

So far we have been dealing with what is called **elastic deformation** of the spring. This happens when the force applied to the spring is directly proportional to the extension and when you remove the force the spring returns to its original length.

A spring will only stretch elastically up to a certain point. This point is called the **elastic limit**. After this limit is reached the deformation is said to be **plastic**.

Plastic deformation means that the force is no longer proportional to the extension and when you remove the force the spring no longer returns to its original length; it has been permanently stretched.

The graph below shows you how to indentify the two different types of deformation.

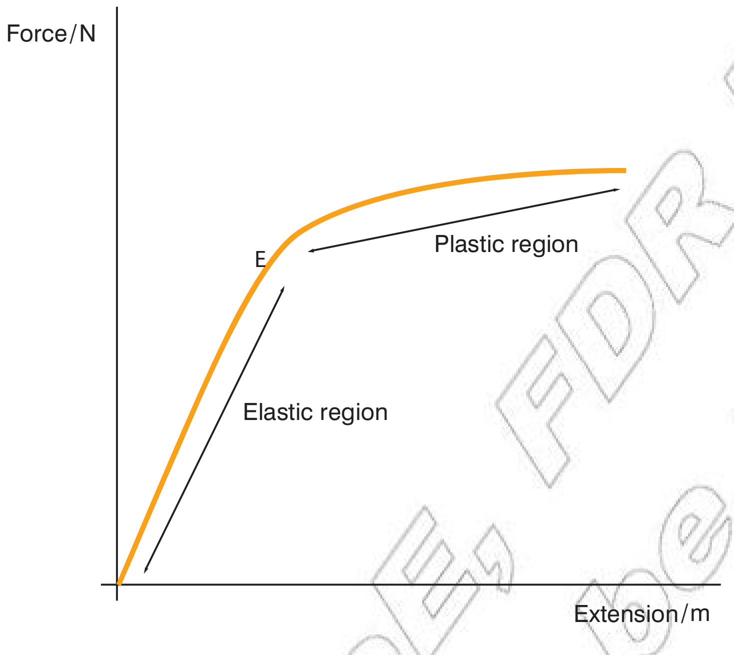


Figure 3.26 *Elastic and plastic deformation of a spring*

E on the graph is the elastic limit. Below the elastic limit the deformation is **elastic**. Above the elastic limit plastic deformation occurs.

Hooke's findings about springs led to the law of elasticity, which is more commonly called Hooke's law. This only applies if the spring is below its elastic limit and so may be written as:

- **The force applied is directly proportional to the extension of a spring up to the elastic limit.**

Different springs have different elastic limits depending on their shape, thickness, material, etc. All materials have an elastic limit; think about a wooden or plastic ruler. If you bend it a little bit it will return to its original length. If you apply too much force it will bend so far it snaps; you've gone beyond the elastic limit for the ruler.

DID YOU KNOW?

The shorthand way of writing directly proportional is to use this symbol: $\propto$. This means we could write Hooke's law as $F \propto \Delta x$ up to the elastic limit.

KEY WORDS

calibrate *to compare a measuring device with a known standard*

elastic deformation *where the force applied is directly proportional to the extension and where the object will return to its original length when the force is removed*

elastic limit *the point up to which a spring will stretch elastically*

plastic deformation *where the force applied is not directly proportional to the extension and where the object will not return to its original length when the force is removed*

Summary

In this section you have learnt that:

- Forces can either be classed as contact or non-contact. Examples of forces include friction, drag, weight, gravitational attraction and contact forces.
- Newton's first law states: "An object will remain at rest or travelling at a constant velocity unless acted upon by an external force".
- Inertia is the tendency of an object to resist changes to its motion. The greater the mass of an object the greater its inertia.
- Hooke's law states: "The force applied to a spring is directly proportional to the extension of the spring up to the elastic limit".
- The stiffer the spring the greater the spring constant (k; measured in N/m).
- Elastic deformation means when forces are removed the object will return to its original length. Plastic deformation means when the forces are removed the object does not return to its original length; it is permanently stretched.

Review questions

1. Give some examples of forces and classify them as contact or non-contact.
2. State Newton's first law and explain what it means.
3. Describe Hooke's law and define the following terms: elastic deformation, elastic limit and plastic deformation.
4. Sketch two force vs. extension graphs, one for a stiff spring the other for a much weaker spring.

3.2 Newton's second law

By the end of this section you should be able to:

- Distinguish between resultant force and equilibrant force.
- Describe the effect of a force acting on a body.
- Apply Newton's second law (as $F_{net} = ma$) to solve problems.
- Resolve forces into rectangular components and compose forces acting on a body using component methods.
- Describe the terms weight and weightlessness (including distinguishing between weight and apparent weight).
- Calculate the weight and apparent weight of an object in a range of situations.

What if more than one force is acting?

There are often several forces acting on an object. As all forces are **vector** quantities we can add them up using the techniques covered in Unit 1.

The overall force acting on any object is referred to as the **resultant force**. This is often called the net force or F_{net}. It is defined as:

- **The vector produced when two or more forces act upon a single object.**

It is calculated by vector addition of the forces acting upon the object.

For example, consider two forces A and B acting on an object. They will produce a resultant force. In the two examples below forces A and B give rise to a resultant, force C.

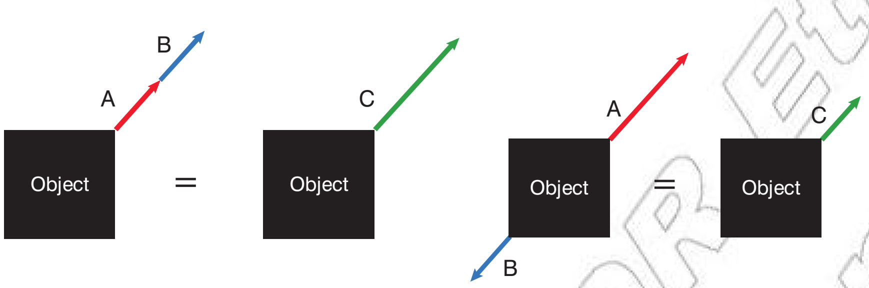


Figure 3.27 *Different resultant forces acting on an object*

If the forces are parallel it is easy to determine the resultant vector. However, if the forces are not parallel (as in Figure 3.28) we then use scale diagrams, parallelogram rules or the mathematical techniques covered in Unit 1 to determine the magnitude and direction of the resultant force.

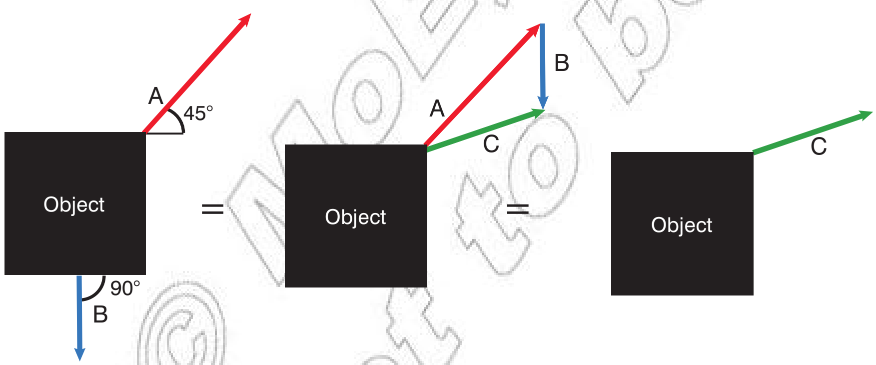


Figure 3.28 *Non-parallel forces leading to a resultant force*

Sometimes it is helpful to know the **equilibrant force**. This is the force you need to apply to a system to cancel out the resultant force. This will result in there being no net force acting on an object.

Figure 3.30 *An equilibrant force will cancel out the resultant force acting on an object.*

KEY WORDS

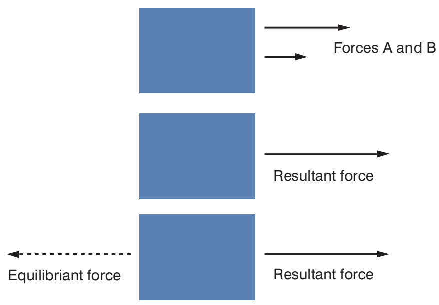
equilibrant force *the force required to cancel out the resultant force*

resultant force *the overall force acting on an object*

DID YOU KNOW?

The equilibrant force for any system is always equal in magnitude to the resultant force but it acts in the opposite direction. This just cancels out the effect of the resultant force. This can be written as: $F_{net} = -F_{equilibrant}$

Figure 3.29 *There are several forces acting on an aircraft in flight.*



Worked example

Two forces are acting on a boat. One force of 400 N is due to current in the river, acting downstream. The other force due to the propeller has a magnitude of 500 N and acts at an angle of 50° to the river bank. Determine the resultant force acting on the boat.

Figure 3.31 *Boat crossing a river*

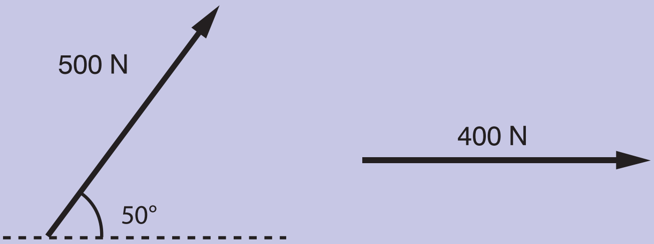


Figure 3.32 *Two force vectors acting on the boat*

We could determine the resultant force using a **scale diagram**. However, on this occasion we are going to find the resultant force mathematically.

In order to determine the resultant force we must first **resolve** the 500 N into horizontal and vertical components using trigonometry.

- Vertical component:
  sin θ = opp / hyp *State principle or equation to be used (trigonometry)*
  hyp x sin θ = opp *Rearrange to make the opp side the subject*
  500 N x sin 50° = 383 N ↑. *Substitute in known values and complete calculation, then clearly state the answer with unit*

- Horizontal component:
  cos θ = adj / hyp *State principle or equation to be used (trigonometry)*
  hyp x sin θ = adj *Rearrange to make the adj side the subject*
  500 N x cos 50° = 321 N →. *Substitute in known values and complete calculation, then clearly state the answer with unit*

We can then add the horizontal forces to give the resultant horizontal force.

- Resultant horizontal force:
  $F_{net\ horizontal}$ = 321 N → + 400 N → *Determine the net horizontal force (note the directions)*
  $F_{net\ horizontal}$ = 721 N →. *Clearly state the answer with unit*

We can then use **Pythagoras's theorem** to determine the magnitude of the resultant force and **trigonometry** to determine the direction.

- Magnitude of resultant force:
  $F_{net\ horizontal}$ = 721 N → *Clearly state known values*
  $F_{net\ vertical}$ = 383 N ↑ *Clearly state known values*
  F_{net} = 383 N ↑ + 721 N → *Determine the net force (note the directions)*
  $F_{net}^{\ 2} = 383^2 + 721^2$ *Apply Pythagoras's theorem*
  $F_{net}^{\ 2}$ = 666 530 *Solve for $F_{net}^{\ 2}$*
  $F_{net} = \sqrt{666\ 530}$ *Rearrange for resultant (take square root) and solve*
  F_{net} = 816 N *Clearly state the answer with unit*

- Direction of resultant force:
  $F_{net\ horizontal}$ = 721 N → *Clearly state known values*
  $F_{net\ vertical}$ = 383 N ↑ *Clearly state known values*
  tan θ = opp / adj *State principle or equation to be used (trigonometry)*
  $\theta = \tan^{-1}$ (opp / adj) *Rearrange equation to make θ the subject*
  $\theta = \tan^{-1}$ (721 / 383) *Substitute in known values and complete calculation*
  θ = 62° *Clearly state the answer with unit*

This is the angle between the resultant and the vertical component. The angle between the resultant force and the river bank is 90°– 62° = 28°.

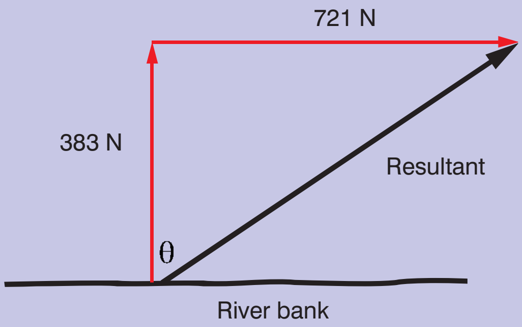


Figure 3.33 Determining the resultant force

In this situation the equilibrant force would be 816 N acting in the opposite direction to the resultant force.

Forces and acceleration

In the previous section we said: Newton's first law means a force is required to make an object:

- **speed up**
- **slow down**
- **change direction.**

If an object does any of these things we can say it is **accelerating**. In other words, forces cause objects to accelerate, or more precisely if there is a resultant force acting on an object, then that object will accelerate. The forces are said to **unbalanced**.

If there are **balanced forces** acting on an object then there is no resultant force and so the object will not accelerate.

KEY WORDS

trigonometry *a type of mathematics that deals with the relationships between the sides and angles of triangles*

accelerating *where an object is speeding up, slowing down or changing direction*

balanced forces *where the forces acting on a body cancel each other out and there is no resultant force*

inversely proportional *a relationship where one variable increases as the other decreases and vice versa*

unbalanced forces *where the forces acting on a body do not cancel out and there is a resultant force*

Figure 3.34 *Any object going around a bend is accelerating; the forces are unbalanced and so there must be a resultant force acting on it.*

Newton's second law relates to the rate of change of momentum of an object (more on this later). He realised that whenever a resultant force acts on an object it will accelerate and this acceleration takes place in the same direction as the force. If you push an object to the left it will accelerate towards the left.

Through careful experiment and investigation he also worked out that if you double the resultant force then the acceleration of the object will also double. In other words the force applied is directly proportional to the acceleration (as long as everything else remains constant).

He also determined that the acceleration of the object also depends on the object's mass. The greater the mass the greater the inertia, and so the lower the acceleration. In fact if you double the mass the acceleration will halve and vice versa. An object with a quarter of the mass will accelerate at four times the rate if the same force is applied. This relationship is called **inversely proportional**. As one quantity doubles the other halves.

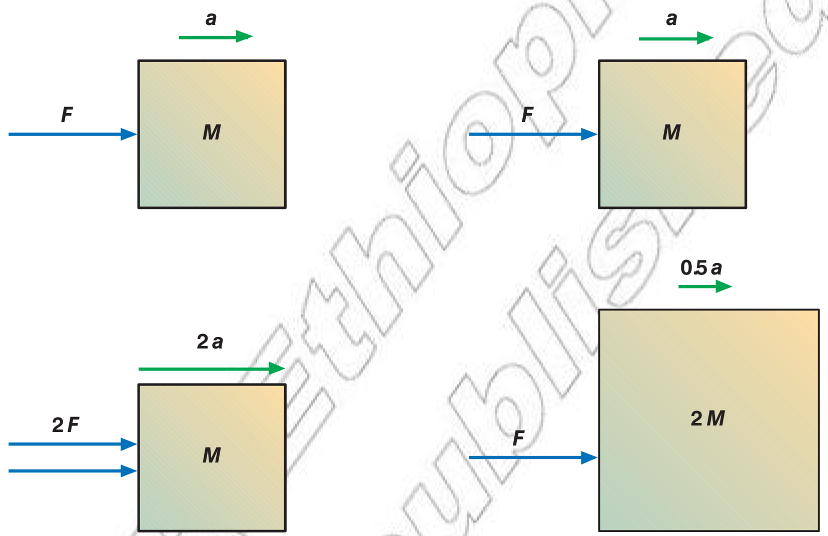


Figure 3.35 *The effects of force and mass on acceleration*

As long as the mass of the object remains constant then Newton's second law can be expressed as:

- **The acceleration of an object is directly proportional to the resultant force acting on the object.**

and

- **This acceleration occurs in the direction of the resultant force.**

(Remember, this only applies if the mass of the object is constant.)

Figure 3.36 *When you apply the brakes on a bike a force is generated in the opposite direction to motion. You accelerate in this direction and so slow down.*

This gives us:

Resultant force = mass of object × acceleration of object

$F_{net} = ma$

We can use this equation to determine the resultant force required to make a car of mass 1200 kg accelerate at 2 m/s^2.

Resultant force = mass of object × acceleration of object *State principle or equation to be used (Newton's second law)*

$F_{net} = ma$ *Simplify statement to symbols*

$F_{net} = 1200\text{ kg} \times 2\text{ m/s}^2$ *Substitute in known values and complete calculation*

$F_{net} = 2400\text{ N}$ *Clearly state the answer with unit*

Figure 3.37 *The greater the force applied to the ball the greater its acceleration.*

We can use the equation to determine the acceleration of a soccer ball if we know the applied resultant force. A footballer may strike a ball of mass 400 g with a force of 200 N.

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$a = F_{net} / m$ *Rearrange equation to make a the subject*

m = 400 g, which is 0.4 kg *Ensure all values are in SI units*

a = 200 N / 0.4 kg *Substitute in known values and complete calculation*

a = 500 m/s² *Clearly state the answer with unit*

This acceleration will be in the same direction as the resultant force.

$F_{net} = ma$ with several forces

If several forces are acting on an object then in order to determine its acceleration we must first determine the resultant force.

To determine the acceleration we would use $F_{net} = ma$.

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$a = F_{net} / m$ *Rearrange equation to make a the subject*

The resultant force in Figure 3.38 is 30 N → *Determine resultant by simple calculation of net force*

a = 30 N / 4.0 kg *Substitute in known values and complete calculation*

a = 7.5 m/s² to the right *Clearly state the answer with unit*

To determine the acceleration we would again use $F_{net} = ma$. Except in this case we must subtract the forces to determine the resultant force.

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$a = F_{net} / m$ *Rearrange equation to make a the subject*

The resultant force in Figure 3.39 is 20 N → *Determine resultant by simple calculation of net force*

- a = 20 N / 2.0 kg *Substitute in known values and complete calculation*
- a = 10 m/s² in the direction of the 50 N force *Clearly state the answer with unit*

This process can be repeated for forces at an angle and for problems involving more than two forces.

If you know the acceleration of the object you can also determine the magnitude and direction of the resultant forces. For example, two people are pushing a 60 kg trolley along. One applies a force of 40 N and the trolley accelerates at 2.0 m/s². Determine the size of the force applied by the other person.

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

F_{net} = 60 kg × 2 m/s² *Substitute in known values and complete calculation*

F_{net} = 120 N *Clearly state the answer with unit*

The resultant force is 120 N.

$F_{net} = F_1 + F_2$ *Express net force in terms of F_1 and F_2*

DID YOU KNOW?

Newton's second law may be used to define the newton as the unit of force. Using $F_{net} = ma$ and making sure the units are all correct (force in N, mass in kg and acceleration in m/s²), we can say that a force 1 N is the force required to give a mass of 1 kg an acceleration of 1 m/s². Or 1 N is equivalent to 1 kg m/s².

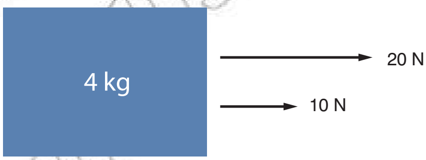


Figure 3.38 *Two forces acting on an object*

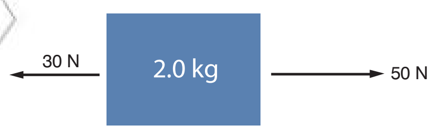


Figure 3.39 *Two forces acting on an object in different directions*

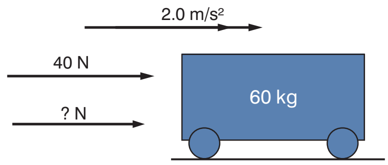


Figure 3.40 *Trolley being pushed by two people*

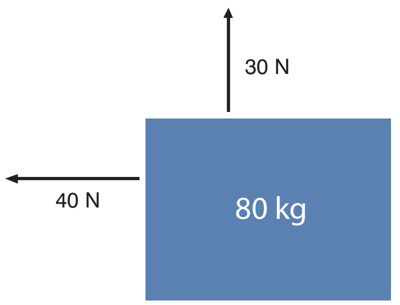


Figure 3.41 Two forces acting on an object at right angles

$120\text{ N} = 40\text{ N} + F_2$ *Substitute in known values and complete calculation*

$F_2 = 80\text{ N} \rightarrow$ *Clearly state the answer with unit*

The same technique may be used to determine the acceleration of an object with two forces acting on it at right angles. For example:

First we must determine the resultant force using Pythagoras's theorem.

$a^2 = b^2 + c^2$ *State principle or equation to be used (Pythagoras's theorem)*

$F_{net}^{\ 2} = (40\text{ N})^2 + (30\text{ N})^2$ *Substitute in known values*

$F_{net}^{\ 2} = 2500$ *Solve for $F_{net}^{\ 2}$ then solve for F_{net} by taking square root*

$F_{net} = 50\text{ N}$ *Clearly state the answer with unit*

Then using $F = ma$ we get:

$a = F_{net} / m$ *Rearrange $F = ma$ to make a the subject*

$a = 50\text{ N} / 80\text{ kg}$ *Substitute in known values and complete calculation*

$a = 0.63\text{ m/s}^2$ *Clearly state the answer with unit*

Trigonometry should then be used to determine the direction of this acceleration; this is in the same direction as the result force (37° to the horizontal – check it for yourself).

Mass and weight

Mass and **weight** are two terms that are frequently confused. We often say we are going to weigh something and then record its mass in kg!

We must make sure we don't muddle the two; they are very different.

Mass is a scalar quantity and it is a measure of the quantity of matter. The more mass the more stuff (the more matter). Remember the ineria of an object depends on its mass, you can think of mass as a measure of an object's inertia. Mass is measured in kilograms (kg).

Weight is a force and so it's a vector quantity, measured in newtons (N). It is the force we experience due to the gravitational pull of the Earth pulling on our mass. Weight is directed towards the centre of the Earth.

We can calculate the weight of an object using:

- weight = mass × gravitational field strength
- $w = mg$

On the surface of the Earth the gravitational field strength is around 9.81 N/kg. We will use 10 N/kg in the following examples to make the mathematics a little easier.

A person with a mass of 70 kg will have a weight of:

$w = mg$ *State principle or equation to be used*

$w = 70\text{ kg} \times 10\text{ N/kg}$ *Substitute in known values and complete calculation*

KEY WORDS

mass *a measure of the quantity of matter*

weight *the force experienced by an object due to the gravitational pull of the Earth*

DID YOU KNOW?

The kilogram is defined as the same amount of mass as the international prototype kilogram. This is a platinum–iridium block held in Paris, France.

Figure 3.42 The international prototype kilogram has a mass of exactly **one** kilogram.

w = 700 N (actually more like 687 N if we use g = 9.81 N/kg).
Clearly state the answer with unit

If the gravitational field strength changes then the weight of the object will change but its mass will stay the same. The gravitational field strength varies a little around the Earth. This is for two reasons.

Firstly the amount of mass between you and the centre of the Earth changes depending on where you are. If there is a particularly dense pocket of material underneath you this will increase the gravitational field strength slightly. The reverse is also true, if there is large pocket of gas or lower density material underneath you the gravitational field strength will go down.

The distance from the centre of the Earth also affects g; it gets smaller the further away from the centre of Earth you get. This change is quite small, you need to move really far away before is becomes noticeable. Even at the top of the tallest mountain g is still around 9.8 N/kg.

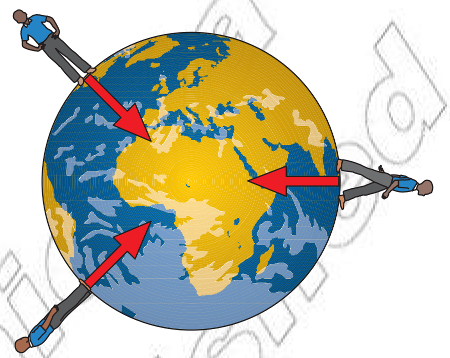


Figure 3.43 *Weight pulls all objects towards the centre of the Earth.*

Remember, only the weight of the object will change; its mass will stay the same. This is also true if we consider different planets. Taking our astronaut as an example, if he stands on the Moon his mass is still 70 kg (there is still the same amount of matter). However, on the Moon the gravitational field strength is much less than that on Earth. This is because the Moon has much less mass and so a weaker gravitational field. The value for g on the moon is just 1.6 N/kg.

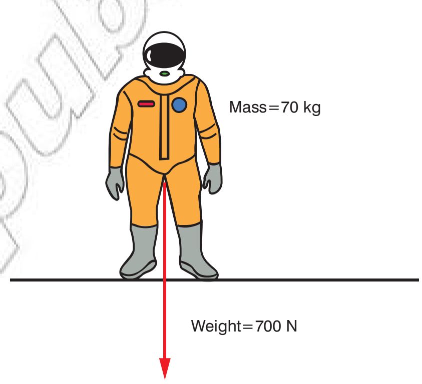


Figure 3.44 *Weight and mass*

His weight on the Moon would be:

$w = mg$ *State principle or equation to be used*

w = 70 kg × 1.6 N/kg *Substitute in known values and complete calculation*

w = 112 N *Clearly state the answer with unit*

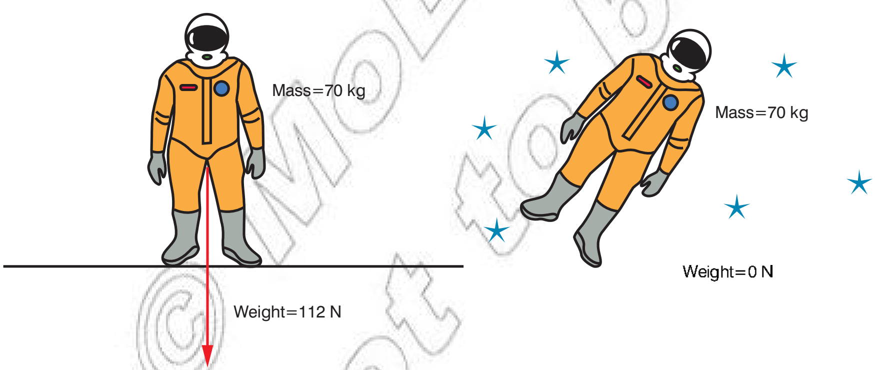


Figure 3.45 *Astronaut on the Moon and in deep space*

In deep space, far away from any planets and stars, the gravitational field strength is pretty much zero. In this case his mass would still be 70 kg. However, his weight would be 0 N; he is weightless.

Think about this...

The value for gravitational field strength is the same value as the acceleration due to gravity (9.81). This can shown by considering an object of mass m dropped from a height above the ground. From Newton's second law we know the acceleration will be equal to $a = F_{net} / m$. We also know the force accelerating the object is the weight of the object so we could write $F_{net} = mg$. Combining these two equations gives us: $a = mg / m$, the m's cancel giving $a = g$!

DID YOU KNOW?

Slight variations in the gravitational field strength are used to look for oil and gas deposits. Because the oil and gas is less dense than the surrounding rock this causes a small dip the gravitational field strength above the deposit. This dip may be detected with sensitive equipment.

Figure 3.47 *Astronauts in the International Space Station are not truly weightless.*

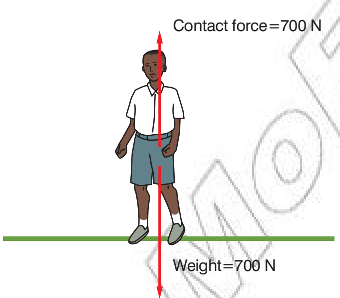


Figure 3.48 *Weight and contact force cancel out*

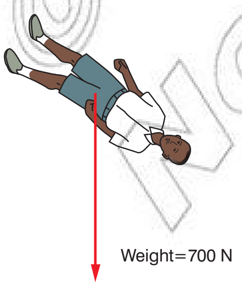


Figure 3.49 *This diver would experience apparent weightlessness for a brief period of time.*

Figure 3.46 *Astronauts on the Moon can carry very large packs due to the Moon's weak gravity.*

True weightlessness and apparent weightlessness

You are only truly weightless if the gravitational field strength is zero. Even astronauts in orbit around the Earth are not truly weightless. There is still a gravitational pull due to the Earth; they still have a weight. So why do they float around?

When we are standing on the ground our weight pulls us vertically downwards towards the centre of the Earth. We push down on the Earth and the Earth pushes back up with a contact force. These two forces cancel out so there is no resultant force (this is why we don't accelerate towards the centre of the Earth; if the ground was not there then we would!).

It is this contact force we feel. We don't notice the pull of gravity. If you take this contact force away by jumping off a tall diving board, our weight accelerates us downwards but we don't feel it. It feels like we are weightless, but we are not!

- **Apparent weightlessness is when the only force acting is your weight.**
- **Real weightlessness is when your weight is zero.**

You get a similar feeling when a car goes over a humpback bridge or when an aircraft climbs or descends. We notice the change in the contact force and this makes us feel like our weight has changed.

Another common example is when you are in a lift. If the lift is not accelerating the two forces are equal, as shown in Figure 3.51.

If the lift accelerates upwards then there must be a net force acting on it. A net force also needs to act on you as you are inside the lift! Imagine the net force acting on you is 200 N (assuming your mass is 70 kg this would give an acceleration of 2.86 m/s^2).

The floor would push you up harder; the contact force would have to increase to 900 N. This provides the extra 200 N. You feel heavier, even though your weight has not changed. It would feel like your weight is 900 N. This is referred to as your **apparent weight**; your **real weight** is still 700 N.

The same is true if the lift were to accelerate downwards. Again imagine the net force on you is 200 N. In this case the contact force would drop 200 N to 500 N. You would feel like your weight has dropped! Your apparent weight would be 500 N.

You can use Newton's second law to determine your apparent weight in an accelerating lift. Taking a person of mass 55 kg then their weight would be:

$w = mg$ *State principle or equation to be used*

$w = 55\ \text{kg} \times 10\ \text{N/kg}$ *Substitute in known values and complete calculation*

$w = 550\ \text{N}$ *Clearly state the answer with unit*

If this person is in a lift accelerating vertically upwards at 2 m/s^2 then the net force acting on the person would be:

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$F_{net} = 55\ \text{kg} \times 2\ \text{m/s}^2$ *Substitute in known values and complete calculation*

$F_{net} = 110\ \text{N}$ *Clearly state the answer with unit*

This force would come from an increase in the contact force. The contact force would have to go up to 660 N (550 N + 110 N). This would be your apparent weight.

Figure 3.50 *The contact force we experience changes dramatically on an exciting roller coaster ride.*

Think about this...

You only notice this effect when the lift accelerates. When the lift is travelling at a steady speed the forces are balanced again (from Newton's first law).

KEY WORDS

apparent weight *the resultant of an object's real weight and any contact forces acting on the object*

real weight *the force experienced by an object solely due to the gravitational pull of the Earth*

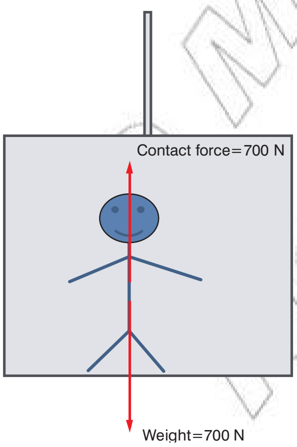


Figure 3.51 *Contact force and weight in a stationary lift.*

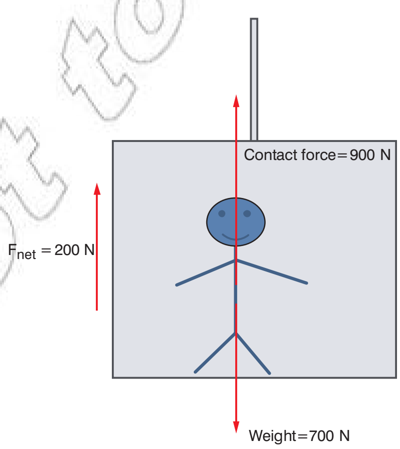


Figure 3.52 *Accelerating upwards requires a net vertical force.*

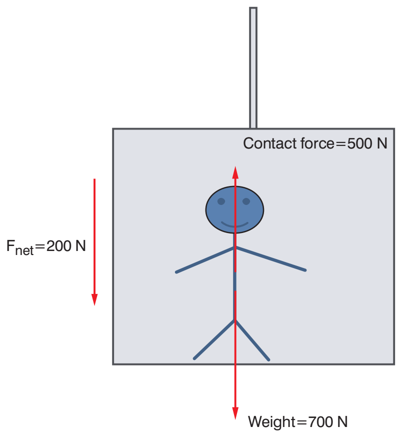


Figure 3.53 *Accelerating downwards requires a net vertical force. This time you would feel like your weight has dropped.*

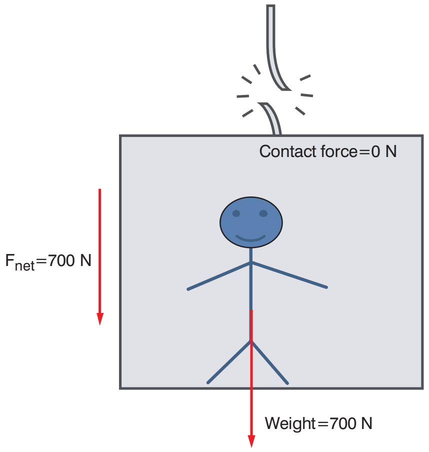


Figure 3.54 *If the contact force is zero you would be apparently weightless.*

DID YOU KNOW?

As part of astronaut training trainees take a flight in an aircraft commonly called the Vomit Comet! This aircraft accelerates downwards at 9.81 m/s^2; this means the contact force inside the aircraft falls to zero. All the occupants become apparently weightless for around 30 s (until the aircraft needs to pull up again).

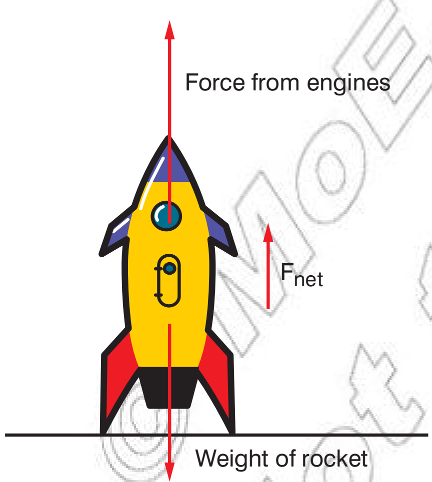


Figure 3.56 *A free body diagram for the forces acting on a rocket at take off.*

If the lift was accelerating downwards at 2 m/s^2 then your apparent weight would be 440 N. This would give a net force vertically downwards equal to 110 N.

If the lift cable were to snap then as the lift accelerates towards the ground the contact force would fall to zero! The floor would stop pushing you up. You would feel like you are weightless. Your apparent weight would be 0 N; you would be apparently weightless.

Figure 3.55 *A photo of the infamous 'Vomit Comet'*

$F_{net} = ma$ considering the weight of the object

We must always think carefully when solving $F_{net} = ma$ problems. Take for example a rocket of mass 15 000 kg. If the engines provide a force of 200 000 N what would its acceleration be?

- $F_{net} = ma$
- $a = F_{net} / m$
- a = 200 000 N / 15 000 kg
- a = 13.3 m/s^2

This is wrong! We've not used the **resultant force**. Remember **free body diagrams** really help to identify all the forces acting on an object.

You can see the resultant force is equal to:

F_{net} = force from engines – weight of rocket *Express F_{net} in terms of all forces acting*

F_{net} = 200 000 N – (15 000 kg × 10 N/kg) *Substitute in known values*

F_{Net} = 200 000 N – 150 000 N *Solve calculation in brackets then complete calculation*

F_{Net} = 5000 N *Clearly state the answer with unit*

This would give us an acceleration equal to:

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$a = F_{net} / m$ *Rearrange equation to make a the subject*

a = 5000 N / 15 000 kg *Substitute in known values and complete calculation*

a = 0.3 m/s^2 *Clearly state the answer with unit*

We must always make sure we consider **all the forces** involved before determining the resultant force acting on an object.

KEY WORDS

free body diagrams *are used to gain an understanding of the forces (or sometimes the fields) acting on an object*

Summary

In this section you have learnt that:

- The overall force acting on an object is called the resultant force. The equilibrant force is the force that needs to be applied to cancel out the resultant force.
- A resultant force will cause an object to accelerate in the same direction as the resultant force.
- Newton's second law states: "Force is directly proportional to acceleration, as long as the mass remains constant, and the acceleration is in the same direction of the force". This gives us $F_{net} = ma$.
- In order to determine the resultant force, the forces acting on the object may need to be resolved then combined together again.
- Mass is a measure of the amount of matter measured in kg, whereas weight is a force measured in N caused by gravity pulling on an object's mass.

DID YOU KNOW?

When large rockets take off their acceleration usually increases for the first few minutes of their flight. The acceleration starts off quite low then increases as the rocket burns fuel. This has a significant effect on its acceleration for two reasons. Firstly the weight drops and so this increases the resultant force acting and secondly as the object has less mass its acceleration will be greater (remember acceleration and mass are inversely proportional).

Review questions

1. Explain what is meant by the terms resultant force and equilibrant force.
2. Describe Newton's second law.
3. Copy and complete Table 3.4.

Table 3.4

Force (N)	Mass (kg)	Acceleration (m/s^2)
100	40	
	60	10
1000		25
	0.2	10
30	600	

4. Figure 3.57 shows the forces acting on three different objects. For each:
   (a) calculate the resultant force acting;
   (b) say whether the forces are balanced or unbalanced;
   (c) calculate the object's acceleration.
5. Explain the differences between mass and weight.

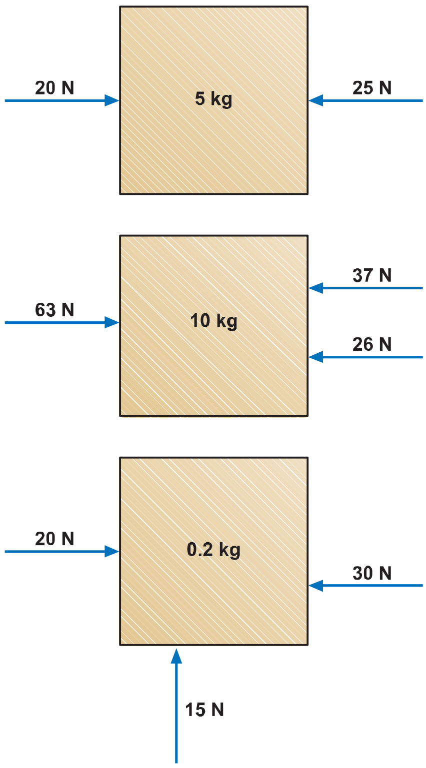


Figure 3.57 *See Question 4*

3.3 Frictional forces

By the end of this section you should be able to:

- Explain the causes of frictional forces.
- Describe the differences between limiting friction, static friction and kinetic friction.
- Draw free body diagrams for objects on inclined planes (to include frictional forces) and use these diagrams to solve problems.

What causes friction?

Friction is a force we experience every single day. Without friction even the simplest of actions, like walking, would be impossible. Friction occurs whenever two solids rub against each other. It is a contact force and it always tends to act in a direction opposing motion.

It is caused by tiny bumps in the surface of the two objects knocking and locking together. No surface is perfectly smooth. This is obvious if you look closely at sandpaper but you need to look really close at smoother objects like a metal sheet.

Figure 3.58 *Even very smooth surfaces have a rough texture at the microscopic level.*

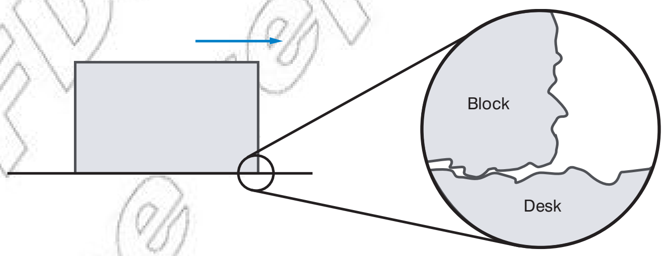


Figure 3.59 *The bumps on the surfaces of material knock together causing friction.*

When magnified, you can see all the small bumps in the surface of a material. It is these bumps that cause friction.

KEY WORDS

friction *the force generated when solids slide or attempt to slide over each other*

Different types of friction

There are two different types of friction. It depends on if the objects in contact are moving or if they are stationary.

Static friction

- **This is the frictional force between two objects that are in contact and trying to move past each other, but not yet moving.**

Imagine gently pushing a heavy book on a desk. At first it does not accelerate. This is because the force you are applying is cancelled out due to static friction. As you gradually increase the force the static friction also increases and the book remains stationary. If

Figure 3.60 *Sandpaper is very rough. Sliding over sandpaper generates a great deal of friction.*

you continue to push harder, eventually the book will slide. The maximum value of the static friction, i.e. the value just before sliding occurs, is called the **limiting friction**.

Kinetic friction (sometimes called dynamic friction)

- **This is the frictional force between two objects sliding over each other.**

It always acts in the opposite direction to motion.

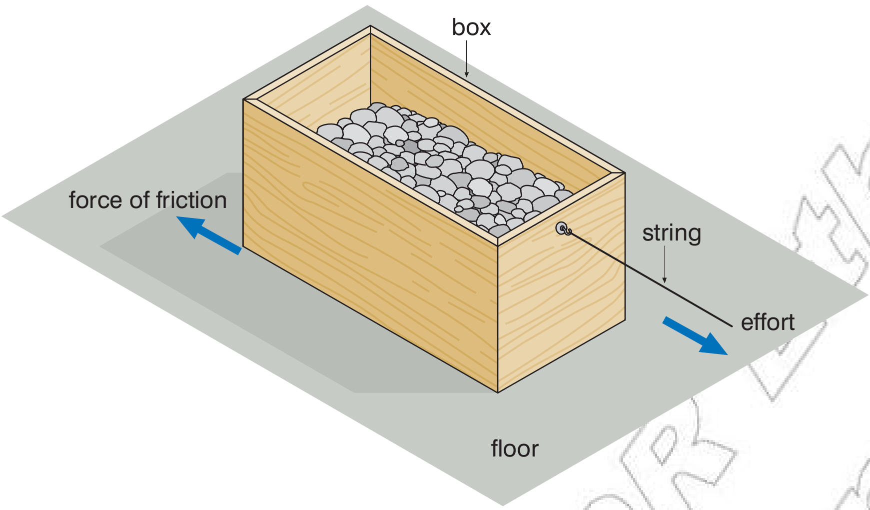


Figure 3.61 *Kinetic friction always acts in the opposite direction to motion.*

The force of friction usually drops when objects start moving and so it is often the case that kinetic friction is less than the limiting friction of a surface.

Factors affecting the frictional force

There are several factors affecting the force of friction between objects.

Perhaps the most obvious is the **roughness** of the surface. The rougher the surface, the greater the friction. In simple terms the bumps on the surface are bigger or more frequent. This causes them to lock together more easily or more often.

You might think the weight of the object affects the friction. A heavier object will push down harder on the surface locking the bumps together harder and so increasing the force of friction. This is generally true but actually it is the contact force that affects the friction. Think about the lift example covered in the previous section. When the lift is accelerating downwards the weight stays the same but the contact force (and so the frictional force) would drop. This is especially important when considering objects on slopes (more on this later).

Think about this...

Friction only happens when solids rub together. This means that there is no such thing as friction with the air or friction through water; both of these examples are types of drag. This is a different type of force.

Figure 3.62 *The friction between snow and ski is very small. This allows professional skiers to reach some very high speeds.*

KEY WORDS

kinetic friction *the frictional force between two objects sliding over each other*

limiting friction *the maximum value of static friction*

static friction *the frictional force between two objects that are trying to move against each other but are not yet moving*

roughness *a measure of the texture of a surface*

DID YOU KNOW?

Friction is really just another example of the electrostatic force. It is caused by the electrons in the atoms in the bumps repelling each other.

Think about this...

The surface area of the objects in contact with each other does not affect the frictional force between them. Although there is a greater area in contact, the weight of the object is more spread out and so there is no change in the frictional force.

KEY WORDS

coefficient of friction *a ratio representing the friction between two surfaces*

The friction force between objects can be calculated using the following equation:

- $F_f = \mu N$

where:

F_f is the frictional force.

μ is a constant called the **coefficient of friction**, which depends on the roughness of the two surfaces. A high coefficient of friction would mean that the surfaces are very rough and so this would lead to a high frictional force. Materials have a static coefficient of friction and a kinetic coefficient of friction, depending on the type of friction being calculated.

N is the normal contact force acting on the block. Normal in this case means at right angles to the surface. If the block is horizontal and there is no vertical acceleration then the normal contact force is equal to the weight.

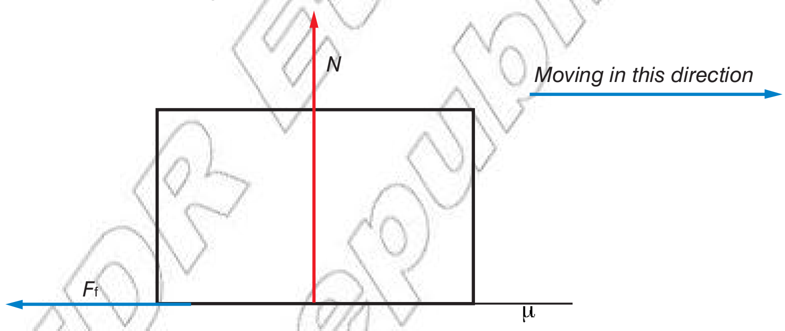


***Figure 3.63** Factors affecting friction*

***Table 3.5** Examples of the static friction coefficient between materials*

Materials rubbing together		μ_{static}
Aluminium	Steel	0.61
Concrete	Rubber	1.00
Concrete	Wood	0.62
Steel	Teflon	0.04
Wood	Wood	0.45

Worked example

The kinetic coefficient of friction between rubber and asphalt is 0.8. Calculate the force of friction acting on a rubber block of mass 2.0 kg as it is pulled along a level road at a steady speed.

$F_f = \mu_{kinetic} N$ *State principle or equation to be used*

$F_f = 0.8 \times N$ *Substitute in known value for $\mu_{kinetic}$*

As the road is level the normal contact force is equal to the weight of the rubber block. In this case the weight = 20 N (2 kg x 10 N/kg)

$F_f = 0.8 \times 20$ N *Substitute in known values and complete calculation*

$F_f = 16$ N *Clearly state the answer with unit*

A 12 kg block of wood is stationary on a horizontal concrete slab. The maximum coefficient of static friction between wood and concrete is 0.65 (this occurs at the limiting friction). What force needs to be applied in order to slide the block along.

$F_f = \mu_{static} N$ *State principle or equation to be used*

$F_f = 0.65 \times N$ *Substitute in known constant for* μ_{static}

As the block is level the normal contact force is equal to the weight of the wood. In this case the weight = 120 N

$F_f = 0.65 \times 120$ N *Substitute in known values and complete calculation*

$F_f = 78$ N *Clearly state the answer with unit*

Activity 3.4: Measuring friction

- Tie a block of wood with string to the hook of a spring balance. Place the block on a table. Pull the balance gradually parallel to the table. Note its reading when the block just starts to move.
- Repeat and take the average of the results.

You have measured the maximum force of static friction (the limiting friction).

- Now pull the balance until the block moves steadily along. Note the reading.
- Repeat several times and take the average.

You have measured the force of dynamic friction.

- Which is greater?

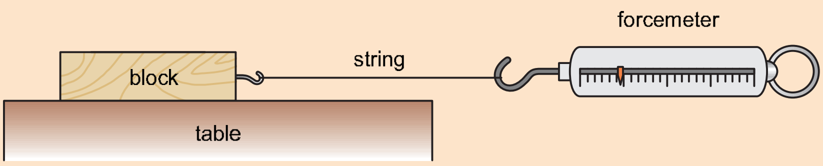


Figure 3.65 *Measuring friction using a spring balance*

You must ensure you pull the block along at a steady speed. This tells us the forces are balanced and the reading on the spring balance is the same as the frictional force.

DID YOU KNOW?

Teflon has one of the lowest coefficients of friction of any material. It was accidentally invented by an American named Roy Plunkett in 1938. The use of Teflon was important in America's development of the atomic bomb. Nowadays its low friction makes it ideal for non-stick frying pans!

Figure 3.64 *Non-stick frying pans have a very low friction coefficient.*

KEY WORDS

inclined plane *a sloping surface or ramp*

Friction and inclined planes

If an object is resting on an **inclined plane** the normal contact force is reduced (the weight stays the same). This means the frictional force is also reduced.

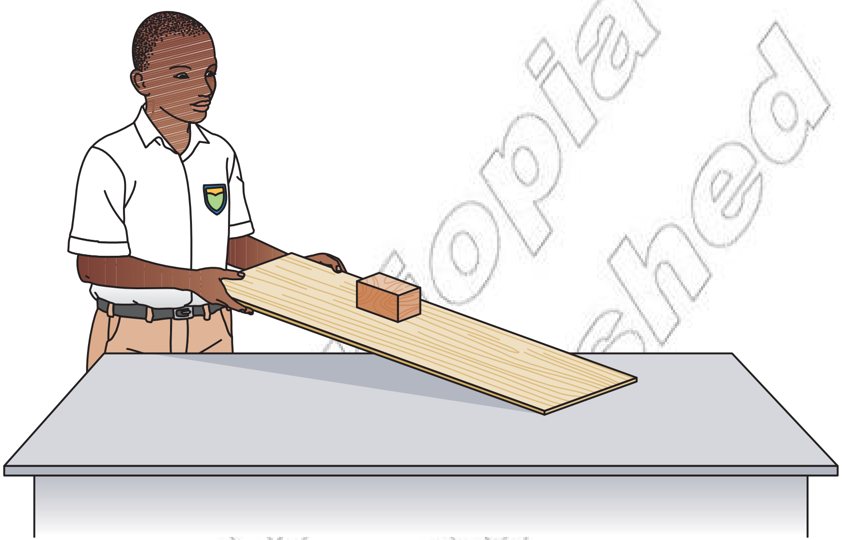


Figure 3.66 *A wooden block on a slope*

Let's assume the block is not sliding down the ramp. If we consider the forces acting on the object we can see that there are three different forces.

As the object is not accelerating (in this case it is stationary) we can conclude from Newton's first law that there is no resultant force acting.

These three forces must form a triangle, as shown in Figure 3.68.

The normal contact force is given by:

- $N = w \cos \theta$

This is always true regardless of if the object is in equilibrium or not. As a result, as the angle of the slope increases the normal contact force falls and so does the force due to friction. If the slope was vertical then the force due to friction would be 0 N.

In order for the block to remain stationary (i.e. the forces remain balanced) then the force due to static friction must equal:

- $F = w \sin \theta$

where w is the weight of the block, F is the force due to static friction and θ is the angle of the slope.

However, the force of friction is also equal to:

- $F_f = \mu_{static} N$

In this case $N = w \cos \theta$, so:

- $F_f = \mu_{static}\, w \cos \theta$

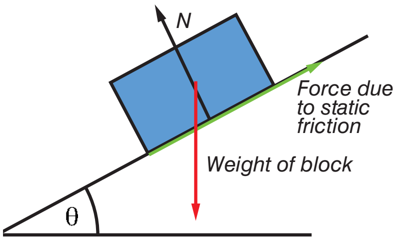


Figure 3.67 *Forces acting on a block on a slope*

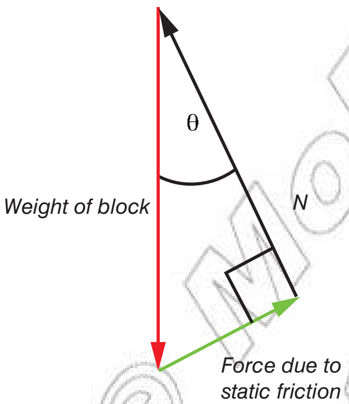


Figure 3.68 *The three forces form a triangle with no resultant force.*

As the angle of the slope increases, cos θ gets smaller. This means the frictional force that can be provided also falls (as all the other variables are constant). At the same time the force required to keep the object stationary ($w \sin \theta$) increases.

This means as the slope gets steeper eventually the block will accelerate down the slope as the forces can no longer be balanced; the limiting friction has been reached and exceeded.

If the object is accelerating down the slope then there must be a resultant force acting on the object.

This resultant force is equal to the difference between $w \sin \theta$ and the force due to kinetic friction.

- $F_{net} = w \sin \theta - \mu_{kinetic} N$

Take, for example, a block of wood of mass 30 kg accelerating down a concrete slope inclined at 45°. We could use the formula above to calculate the acceleration of the block. The $\mu_{kinetic}$ between the wood and the slope is = 0.45.

First we need to find the resultant force:

- $F_{net} = w \sin \theta - \mu_{kinetic} N$ *Express F_{net} in terms of other forces*

In this case the weight of the block is 300 N (from $w = mg$) and the normal contact force is 212 N (from $N = w \cos \theta$).

F_{net} = 300 N × sin 45° – (0.45 × 212 N) *Substitute in known values and complete calculation*

F_{net} =117 N *Clearly state the answer with unit*

The acceleration of the block can then be calculated using Newton's second law.

$F_{net} = ma$ *State principle or equation to be used (Newton's second law)*

$a = F_{net} / m$ *Rearrange equation to make a the subject*

a = 117 N / 30 kg *Substitute in known values and complete calculation*

a = 3.3 m/s^2 *Clearly state the answer with unit*

Reducing friction

In order to reduce the friction between objects there are two techniques that can be used.

Polishing

Polishing or sanding down an object reduces the size of the bumps on the surface. This makes it smoother and so the coefficient of friction drops.

Figure 3.70 *Polishing reduces the roughness of a surface.*

Think about this...

Putting the two equations for the friction force equal to each other: $w \sin \theta = \mu_{static} w \cos \theta$. This can be rewritten as $\tan \theta = \mu_{static}$ and so the maximum angle of the slope before the block will slide is given by $\theta = \tan^{-1} \mu_{static}$. The higher the maximum value for μ_{static} the steeper the slope can before the object slides down the slope.

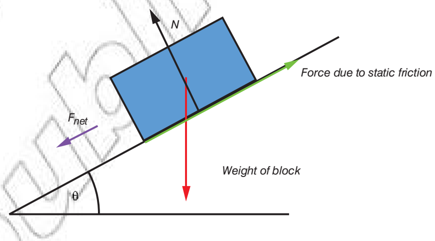


Figure 3.69 *The object slides due to a resultant force.*

Figure 3.72 Without oil vehicle engines would heat up too much and possibly seize up entirely.

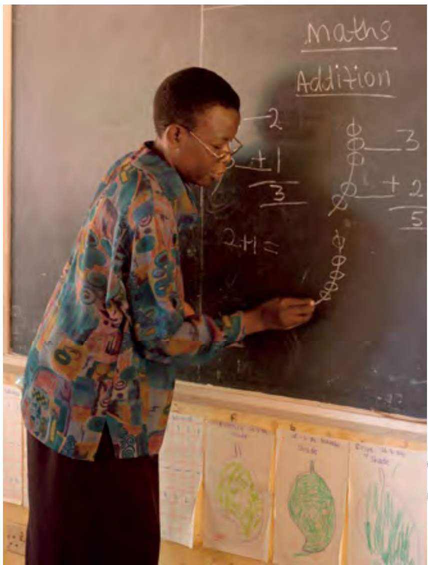


Figure 3.73 Friction between board and chalk causes the chalk to gradually wear away

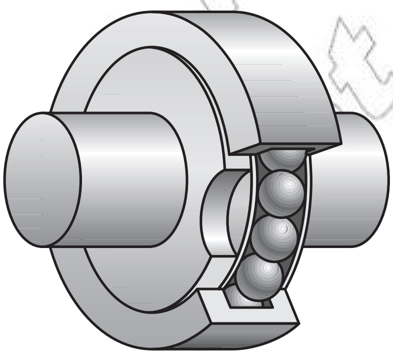


Figure 3.74 Ball bearings ensure that a wheel turns smoothly on its axle.

Lubrication

Lubricating between the surfaces rubbing together also reduces friction. Commonly used lubricants include oil, water and even graphite.

The lubricant effectively fills the gaps between the materials, preventing them from bumping into each other and so allowing them to slide over each other easily.

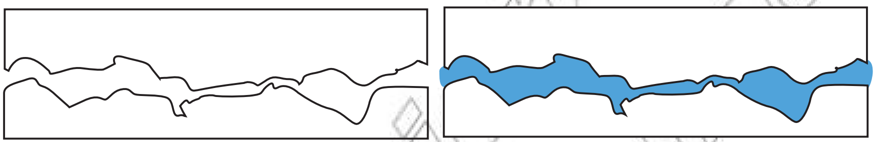


Figure 3.71 Lubrication keeps the surfaces apart.

The effects of friction

Friction causes a heating effect. When you rub your hands together friction between them causes them to warm up. This has many applications but also causes several problems.

Advantages of friction

Is friction always a problem? No. We could not walk if there was no friction. Our feet would slip, just as they do on ice, banana skin or very smooth surfaces. Rubber-soled shoes and car tyres have 'tread' on them to increase friction. Smooth tyres tend to skid, especially on wet, greasy or icy roads.

The brakes on a bicycle, car or other vehicle make use of friction. The brake pads press on the wheels, slowing them down.

Figure 3.73 shows one situation where friction is useful. Without friction, the teacher's chalk would not mark the board.

Disadvantages of friction

When two parts of any machine rub against each other, the friction between them causes heat, noise and wear. The heat produced in fast-moving machines may be so great that the parts become red-hot.

Friction is reduced by lubrication with grease, oil or graphite. Bicycles and sewing machines need oil regularly. The engine of a motor car has a case at the bottom, called a sump, which is full of oil. This covers all the moving parts in the engine. If the engine has too little oil, the pistons and cylinders become so hot that they join together.

A bicycle wheel must turn freely. If there is friction between the wheel and its axle, the bicycle will be harder to ride. Ball bearings between the wheel and axle allow the wheel to turn freely – see Figure 3.74.

Summary

In this section you have learnt that:

- Friction is a force generated when solids either attempt to slide or slide over each other.
- Friction is caused by bumps in the surface of the materials.
- Static friction occurs when objects try to move past each other. Kinetic friction occurs when objects slide over each other; it acts in the opposite direction to motion.
- Frictional forces can be calculated using $F = \mu N$ (where N is the normal contact force – this reduces if the object is on an inclined plane).

Review questions

1. Describe the causes of friction and the factors that affect it.
2. Explain the difference between static friction and kinetic friction.
3. If the static friction between wood and concrete is 0.62, determine the force required to make a wooden block of mass 2 kg start to slide.
4. Give two examples in which friction is useful and two where it is a disadvantage.

3.4 Newton's third law

By the end of this section you should be able to:

- State Newton's third law.
- Describe experiments to demonstrate it and give examples of where it is applicable.

The third law of motion

Newton's third law deals with what happens when you apply a force. It is perhaps the most counter-intuitive of Newton's three laws. It states:

- **If body A exerts a force on body B then body B will exert an equal and opposite force on body A.**

In simple terms this means whenever you push an object it pushes back with an equal and opposite force; essentially forces come in pairs. You can't apply a force to an object without that object applying the same force back onto you.

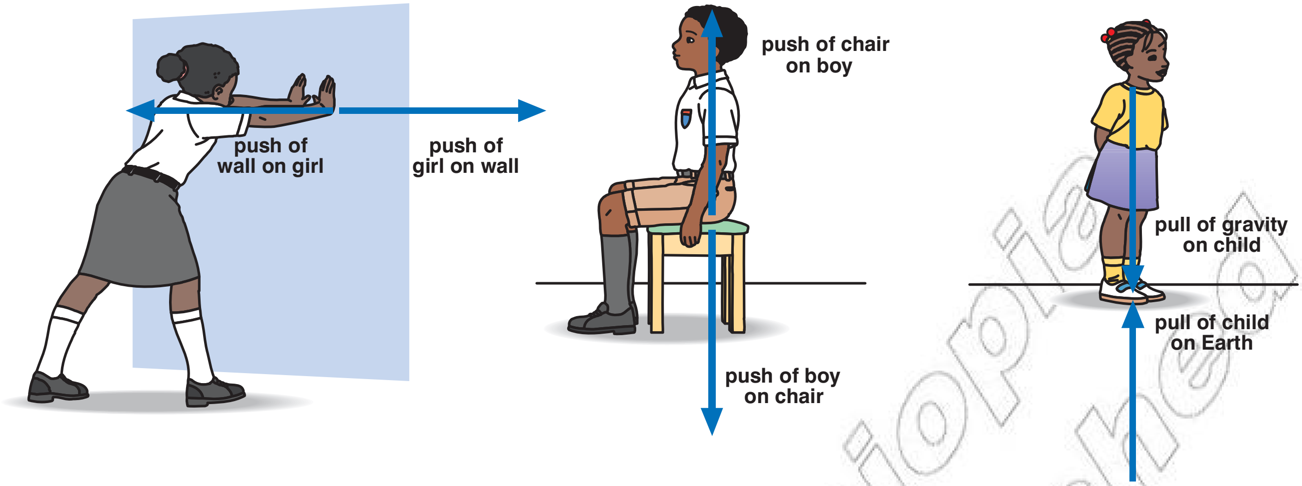


Figure 3.75 *Examples of Newton's third law in action.*

Figure 3.76 *An example of Newton's pairs*

KEY WORDS

Newton's pair *a pair of equal and opposite forces acting between two objects*

The pairs of forces are often called a **Newton's pair** or an action and reaction pair. It is important to notice that they are equal and opposite.

- **Equal: same magnitude**
- **Opposite: opposite direction**

If you push **down** on the desk with a force of **10 N** the desk pushes back **up** with a force of **10 N**. This applies to all forces!

It may seem strange but the gravitational attraction of the Earth on a satellite is exactly the same size as the pull on the Earth from the satellite. The same is true at ground level. If you hold a stone above the Earth then it pulls the Earth up with the same force that the Earth pulls to stone down. When you drop it the stone appears to fall but both the stone and the Earth experience the same force. However, the stone's acceleration is much, much greater as it has much less mass.

To correctly identify Newton's pairs it is worth remembering that the pairs of forces must fit the following four criteria:

- **equal in magnitude**
- **opposite direction**
- **act on different bodies**
- **same type of force**

So, for example, consider a book on a desk.

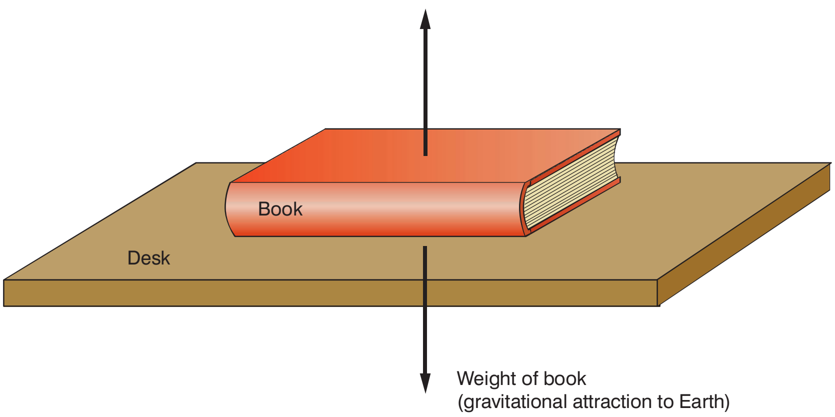


Figure 3.77 *Two forces acting on a book, but they are not a Newton pair.*

Figure 3.77 shows two forces acting on the book, but they are not an action–reaction pair. They are equal and opposite but they do not act on different bodies and they are not the same type of force.

So, where are the Newton's pairs in this example?

Table 3.6 *Newton's pairs for a book on a desk*

Force	**Newton's pair**
Contact force on book from desk	Contact force on desk from book
Weight of book (gravitational attraction of the Earth pulling on the book)	Gravitational attraction of the book pulling on the Earth

The book pushes down on the desk and pulls the Earth upward due to gravitational attraction. These are the pairs to the two forces in Figure 3.77. If we draw three free body diagrams (Figure 3.78) we can more easily see the pairs of forces.

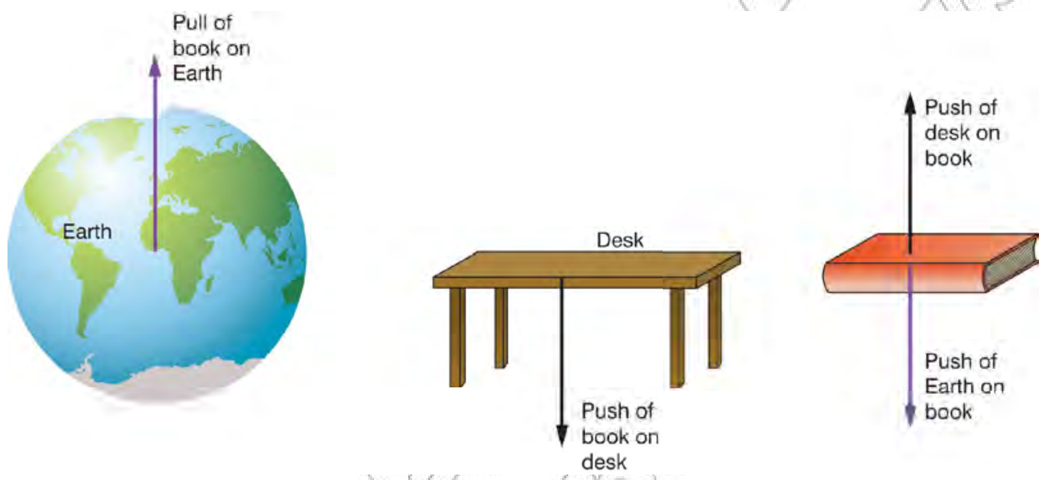


Figure 3.78 *The two pairs of forces*

There are two more pairs of forces not included in Figure 3.78. Can you work out what they are? (Hint: they do not involve the book).

Applications of the third law

Newton's third law is incredibly important to motion. Applications such as rockets, jet engines, cars and even just walking around rely on this law.

When you walk you push backwards on the ground; at the same time the ground pushes forward on you and so you accelerate forwards! The same is true with car tyres.

With a rocket or jet engine hot gases are blasted out of the back; they are in essence pushed out. This results in an equal and opposite force on the engine pushing it forward.

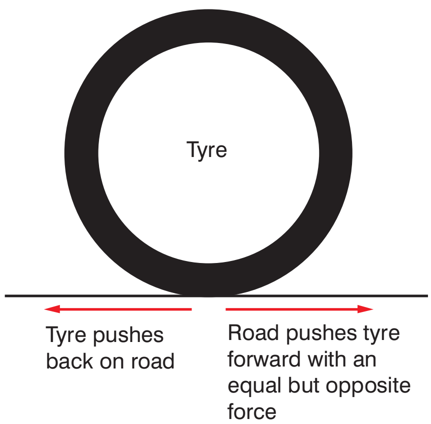


Figure 3.79 *Newton's third law in action*

Figure 3.80 *Without Newton's third law space rockets would not be able to move!*

Activity 3.5: Discovering equal and opposite forces

With a partner get a rope and two skateboards. Both stand on a board some distance apart and hold a rope between you. If one of you holds the rope and the other one pulls it towards them, who moves?

You both will! An equal and opposite force is exerted on the puller. If he pulls with twice as much force he will experience twice as much force pulling him forwards.

Summary

In this section you have learnt that:

- Newton's third law states: "If body A exerts a force on body B then body B will exert an equal and opposite force on body A".
- Newton's third law means forces always come in pairs.

Review questions

1. State Newton's third law.
2. Describe the characteristics of Newton's pairs of forces and give three different examples.

DID YOU KNOW?

As well as linear momentum there is another physical property called **angular momentum**. This is all to do with the rotation of a spinning object. For the purpose of the following sections any reference to momentum refers to linear momentum.

3.5 Conservation of linear momentum

By the end of this section you should be able to:

- Define linear momentum and state its units.
- State the law of conservation of momentum.
- Define the term impulse and state its units.
- Solve numerical problems relating to momentum, conservation of momentum and impulse.
- State Newton's second law in terms of momentum.

What is linear momentum?

Linear momentum is another important idea in physics. It can be thought of as a measure of how hard it is to stop a moving object; the 'unstopability' of the object. Objects with a larger linear momentum are harder to stop!

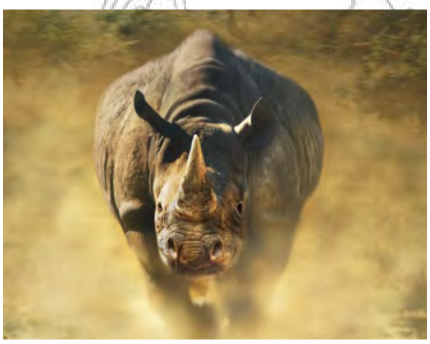


Figure 3.81 *A charging rhino has a large momentum!*

There are two factors that make an object hard to stop, its **mass** and its **velocity**. The greater the mass the harder it is to stop, the faster an object is moving the harder it is to stop. Linear momentum is defined as the product of an object's mass and velocity. This leads to the equation for linear momentum:

- **linear momentum = mass × velocity**

Or in symbols:

- $p = mv$

p is the symbol for linear momentum and as the units of mass are kg and the units of velocity are m/s it follows the units of linear momentum are kg m/s.

For example, a rhino running at top speed has quite a large momentum; it's very hard to stop! An adult black rhino may have a mass of 1000 kg and for short periods of time can reach 15 m/s when sprinting. To find its momentum we would use the equation:

momentum = mass × velocity *State principle or equation to be used (definition of momentum)*

p = 1000 kg × 15 m/s *Substitute in known values and complete calculation*

p = 15 000 kg m/s *Clearly state the answer with unit*

A sprinting human may have a momentum of around 640 kg m/s (assuming a velocity of 8 m/s and a mass of 80 kg).

Momentum is a **vector** quantity. This means the direction of motion of the object is really important. For example, take a situation where two identical cars are heading towards each other.

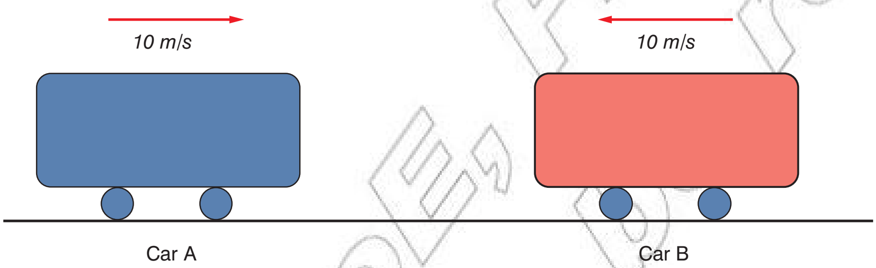


Figure 3.83 *Two head-on cars*

The momentum of car A is:

$momentum_A = mass_A \times velocity_A$ *State principle or equation to be used (definition of momentum applied to car A)*

p_A = 1200 kg × 10 m/s *Substitute in known values and complete calculation*

p_A = 12 000 kg m/s to the right *Clearly state the answer with unit*

The momentum of car B is:

$momentum_B = mass_B \times velocity_B$ *State principle or equation to be used (definition of momentum applied to car B)*

p_B = 1200 kg × 10 m/s *Substitute in known values and complete calculation*

p_B = 12 000 kg m/s to the left *Clearly state the answer with unit*

KEY WORDS

angular momentum *the momentum of an object moving in a circle*

linear momentum *a measure of how hard it is to stop a moving object*

law of conservation of linear momentum *law stating that in a closed system, the total linear momentum will remain constant*

Figure 3.82 *Due to its large mass a moving train has a large momentum.*

Think about this...

The equation for momentum shows that both the mass and velocity of an object are directly proportional to its momentum. This means an object with twice the mass travelling at the same speed will have double the momentum. Alternatively, an object going twice as fast will have double the momentum.

This could be written as:

$p_B = -12\ 000$ kg m/s.

This is really important because if you consider the cars together as one system then the total momentum is 0 kg m/s (not 24 000 kg m/s).

DID YOU KNOW?

There are several conservation laws in physics. They include conservation of energy, conservation of linear momentum, conservation of angular momentum and conservation of electric charge. They form the basis of modern physics and always apply!

The law of conservation of linear momentum

One of the most important conservation laws in physics is the **law of conservation of linear momentum**. It states:

- **In a closed system the total linear momentum must remain constant.**

This means that when objects collide the total linear momentum before the collision must equal the total linear momentum after the collision *as long as no external forces act on the system*. In symbolic terms this may be written as:

- $\Sigma p_{initial} = \Sigma p_{final}$

Σ means 'sum of'.

Take, for example, a ball of mass 2.0 kg travelling at 5 m/s towards a ball of mass 1 kg.

Figure 3.84 *Two balls about to collide*

The momentum before the collision must equal the momentum of ball A plus the momentum of ball B. Ball B is not moving so it has a momentum of 0 kg m/s.

$\text{momentum}_A = \text{mass}_A \times \text{velocity}_A$ *State principle or equation to be used (definition of momentum)*

$p_A = 1.0 \text{ kg} \times 5 \text{ m/s}$ *Substitute in known values and complete calculation*

$p_A = 5$ kg m/s to the right *Clearly state the answer with unit*

This is the total momentum before the collision. The law of conservation of momentum states the momentum after the collision must also equal 5 kg m/s to the right.

This gives us several possible outcomes.

Outcome 1: Ball A stops and ball B moves away with a certain velocity.

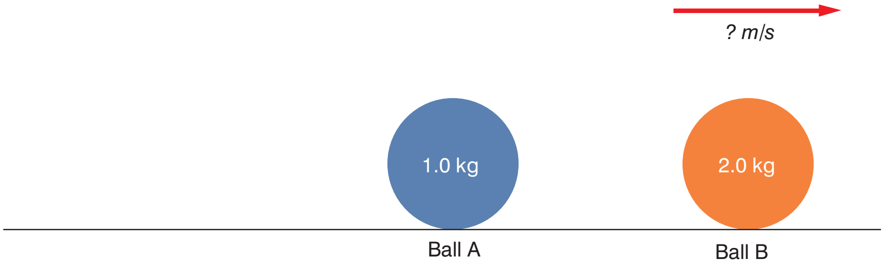


***Figure 3.85** Ball B moves away from ball A*

We can work out the velocity of ball B. As the total momentum of the system must equal 5 kg m/s then the momentum of ball B must be 5 kg m/s.

$\text{momentum}_B = \text{mass}_B \times \text{velocity}_B$ *State principle or equation to be used (definition of momentum)*

$\text{velocity}_B = \text{momentum}_B / \text{mass}_B$ *Rearrange equation to make velocity_B the subject*

$v_B = 5$ kg m/s / 2.0 kg *Substitute in known values and complete calculation*

$v_B = 2.5$ m/s to the right *Clearly state the answer with unit*

Thinking about this answer it makes sense. Ball B has twice the mass of ball A and so the velocity will need to be half of that of ball A before they collided.

Outcome 2: The balls stick together (imagine there are magnets inside them) and they move away together with a certain velocity.

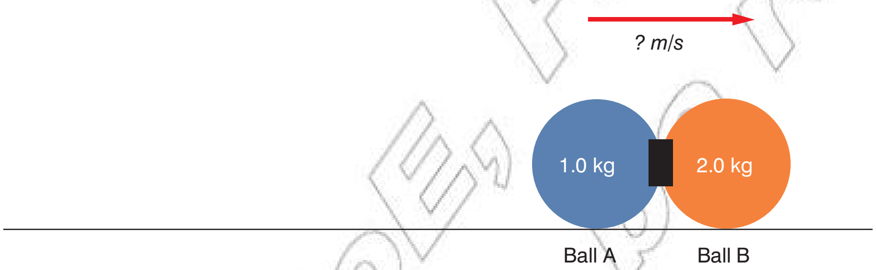


***Figure 3.86** The balls stick together*

We can work out the velocity of the balls when they stick together. Just like the previous example the total momentum of the system must equal 5 kg m/s then the momentum of the balls must be 5 kg m/s.

momentum = mass × velocity *State principle or equation to be used (definition of momentum)*

velocity = momentum / mass *Rearrange equation to make velocity the subject*

$v = 5$ kg m/s / 3.0 kg *Substitute in known values and complete calculation*

Notice we had to use a mass of 3.0 kg as this is the total mass of the two balls.

- $v = 1.7$ m/s to the right *Clearly state the answer with unit*

Think about this...

If the mass of the system remains constant we can rewrite the equation as $F_{net} = m\Delta v / \Delta t$. Only velocity is changing as the mass is constant. From Unit 2 we know that $\Delta v / \Delta t$ is the acceleration of the object. As a result we get $F_{net} = ma$ but only if the mass is constant!

Think about this...

If the object changes direction then you mustn't forget momentum is a vector quantity. A ball going from a momentum of 10 kg m/s to the left to 5 kg m/s to the right has experienced a change of momentum of 15 kg m/s to the right.

Outcome 3: Ball A bounces back off ball B. Ball B moves to the right with a velocity of 3 m/s and ball A moves back in opposite direction.

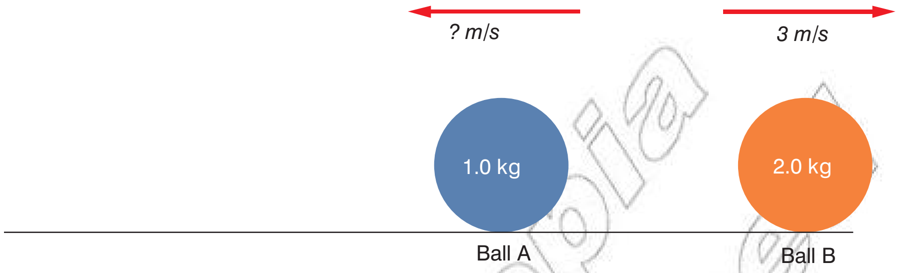


***Figure 3.87** Ball A bounces off ball B and both balls move*

Again, just like the previous example the total momentum of the system must equal 5 kg m/s. However, this time both the balls have a momentum. The momentum of ball B is given by:

$\text{momentum}_B = \text{mass}_B \times \text{velocity}_B$ *State principle or equation to be used (definition of momentum)*

$p_B = 2.0\text{ kg} \times 3\text{ m/s}$ *Substitute in known values and complete calculation*

$p_B = 6\text{ kg m/s}$ to the right *Clearly state the answer with unit*

In order for momentum to be conserved ball A must have a momentum of –1 kg m/s or a momentum of 1 kg m/s to the left. This will give us a total momentum of 5 kg m/s to the right.

The velocity of ball A can then be calculated.

$\text{momentum}_A = \text{mass}_A \times \text{velocity}_A$ *State principle or equation to be used (definition of momentum)*

$\text{velocity}_A = \text{momentum}_A / \text{mass}_A$ *Rearrange equation to make velocity_A the subject*

$v_A = -1\text{ kg m/s} / 1.0\text{ kg}$ *Substitute in known values and complete calculation*

$v_A = -1\text{ m/s}$ or 1 m/s to the left *Clearly state the answer with unit*

There are several other possible outcomes depending on the masses of the objects and the materials they are made out of. **In every possible case the linear momentum of the system must be conserved!**

Explosions

When a gun is fired, an explosion occurs inside the gun and the bullet flies off at high speed. The person firing the gun has to be ready for the recoil – the gun pushes back against their shoulder, in the opposite direction to the direction of the bullet. Figure 3.88 shows why this is.

- The bullet has a small mass and a high velocity, towards the right.
- The gun has a larger mass and a smaller velocity, towards the left.

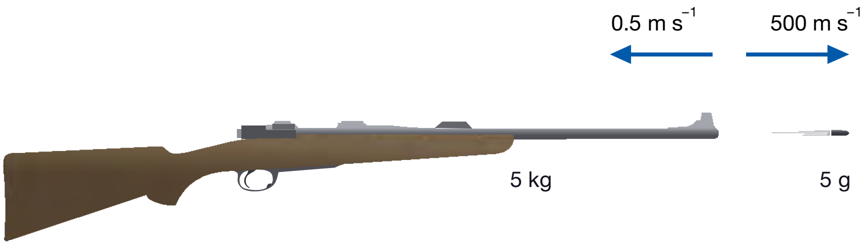


Figure 3.88 *The momentum of the bullet is equal and opposite to the momentum of the gun*

Before the explosion, neither the gun nor the bullet had any momentum. In the explosion, the bullet is given momentum to the right, while the gun is given an equal amount of momentum to the left. Recall that momentum is a vector quantity; equal and opposite amounts of momentum cancel out, so the total amount of momentum after the explosion is zero. Hence there is just as much momentum after the explosion as there was before it, so we can again see that momentum has been conserved.

Back to Newton's second law

Earlier we discussed Newton's second law as:

- **The acceleration of an object is directly proportional to the resultant force acting on the object.**

and

- **This acceleration occurs in the direction of the resultant force.**

However, this only applies if the mass of the system remains constant. Newton's original concept for the second law involved forces changing the linear momentum of objects.

He said:

- **The resultant force acting on an object must be directly proportional to the rate of change of linear momentum of the object.**

and

- **The change in linear momentum occurs in the same direction as the resultant force.**

Using symbols this becomes:

- $F_{net} = \Delta^{mv} / \Delta t$

(Remember the Δ means 'change in'.)

To recap, the law of conservation of linear momentum states that the momentum must remain constant unless an external force acts. What Newton's second law tells us is that the momentum of a system can change if a force acts on it. The two compliment each other!

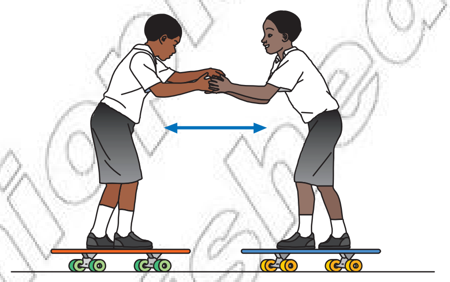


Figure 3.89 *An explosive situation*

Activity 3.6: The human explosion

- Find two students with the same mass. Make them stand on platforms with wheels, facing each other (Figure 3.89).
- One student pushes the other gently, in an attempt to make him or her move away. (This is a simple way of making an 'explosion' in the lab.) What happens?
- Does it make any difference which student does the pushing, or if both push?
- Try again with students having different masses.

KEY WORDS

impulse *the magnitude of a force multiplied by the time for which it acts*

Worked example

A car of mass 1400 kg accelerates from 10 m/s to 15 m/s over 3.5 s. Find the average resultant force acting.

$F_{net} = \Delta^{mv} / \Delta t$ *State principle or equation to be used (Newton's second law in terms of momentum)*

The change in momentum is equal to the final momentum minus the initial momentum.

$\Delta mv = mv - mu$ *Express simple statement of change in momentum*

Δmv = (1400 kg × 15 m/s) – (1400 kg × 10 m/s) *Substitute in known values and complete calculation*

Δmv = 7000 kg m/s *Clearly state the answer with unit*

$F_{net} = \Delta mv / \Delta t$ *State principle or equation to be used (Newton's second law in terms of momentum)*

F_{net} = 7000 kg m/s / 3.5 s *Substitute in known values and complete calculation*

F_{net} = 2000 N, in the direction of its acceleration *Clearly state the answer with unit*

(As the mass of this system can be assumed to be constant we could have used $F_{net} = ma$).

Worked example

Imagine gently hitting a tennis ball of mass 100 g with a force of 50 N. The tennis racket and ball are in contact for just 0.02 s. We can calculate the change in momentum.

$F_{net} = \Delta mv / \Delta t$ *State principle or equation to be used (Newton's second law in terms of momentum)*

$\Delta mv = F_{net} \times \Delta t$ *Rearrange equation to make Δmv the subject*

Δmv = 50 N × 0.02 s *Substitute in known values and complete calculation*

Δmv = 1.0 kg m/s in the direction of the 50 N force *Clearly state the answer with unit*

Acting on impulse

The **impulse** of a force is the magnitude of the force multiplied by the time which it acts.

- Impulse = $F\Delta t$

The units of impulse are usually expressed as **N s**.

An impulse of 10 N s could be caused by a 10 N force acting for 1 s or a 1 N force acting for 10 s (and thousands of other combinations!).

From Newton's second law we get:

- $F_{net} = \Delta mv / \Delta t$

This can be written as:

- $F_{net} \Delta t = \Delta mv =$ impulse
- **The impulse of a force is also equal to the change in momentum of the object.**

So, the longer the force acts on an object the greater the impulse and so the greater the change in momentum.

Think about this...

In most sports participants are encouraged to follow through when kicking or hitting a ball. This increases the time the force is acting and so gives rise to a greater impulse and so a greater change in momentum.

Worked example

A footballer kicks a stationary ball of mass 1 kg with a force of 90 N. The first time his foot is in contact with the ball for just 0.01 s. The second time he follows through and his foot is in contact with the ball for 0.1 s. Find the impulse, change in momentum and the velocity of the ball after impact in each case.

Table 3.7 *Calculating the velocity of footballs*

Δt = 0.01 s	**Δt = 0.1 s**
Impulse = $F\Delta t$	Impulse = $F\Delta t$
Impulse = 90 N × 0.01 s	Impulse = 90 N × 0.1 s
Impulse = 0.9 N s	Impulse = 9 N s
Change in momentum = impulse	Change in momentum = impulse
Change in momentum = 0.9 kg m/s	Change in momentum = 9 kg m/s
As the initial momentum was 0 kg m/s the change in momentum must equal the final momentum of the ball.	As the initial momentum was 0 kg m/s the change in momentum must equal the final momentum of the ball.
Final momentum = 0.9 kg m/s	Final momentum = 9 kg m/s
$p = mv$ so $v = p / m$	$p = mv$ so $v = p / m$
v = 0.9 kg m/s / 1 kg	v = 9 kg m/s / 1 kg
v = 0.9 m/s	v = 9 m/s

Newton's laws and conservation of linear momentum

Using Newton's laws we can prove the law of conservation of linear momentum. Imagine two railway carriages. If one crashes into the other, they will exert equal and opposite forces on each other (Newton's third law). This force will be acting for the same time on each carriage; therefore the impulse on each carriage will be the same ($F\Delta t$).

Both carriages will experience the same force but in opposite directions. They will therefore have the same change in momentum, but in opposite directions (Newton's second law).

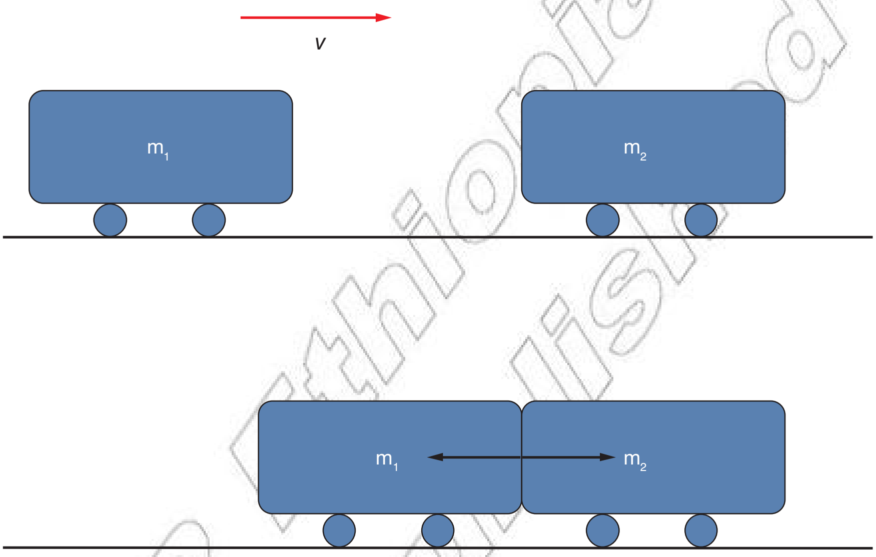


Figure 3.90 *Two railway carriages colliding*

The change in momentum of each carriage is given by $F\Delta t = \Delta mv$. The first carriage will experience a change in momentum equal and opposite to the second carriage. Therefore:

- $m_1\Delta\, v_1\Delta = -m_2\, \Delta v_2$

Or

- $0 = m_1\Delta\, v_1 - m_1\Delta\, v_1$

The total change of momentum of the system is 0 kg m/s; therefore the momentum has not changed and momentum has to be conserved!

Summary

In this section you have learnt that:

- Linear momentum is defined as the product of an object's mass and velocity (as given by $p = mv$). It is a vector quantity measured in kg m/s.
- The law of conservation of momentum states: "In a closed system the total linear momentum remains constant." This means if there are no external forces acting then the total momentum before a collision/explosion must be the same as the total momentum after the collision/explosion.
- The impulse of a force is defined as the force multiplied by the time the force is acting. It has units of N s. Impulse is equal to the change in momentum of an object.

- In terms of momentum, Newton's second law can be written as: "The resultant force acting on an object must be directly proportional to the rate of change of linear momentum of the object and the change in linear momentum occurs in the same direction as the resultant force." Using symbols this becomes: $F_{net} = \Delta mv / \Delta t$

Review questions

1. Define linear momentum and state its units.
2. Calculate the momentum of a car of mass 1200 kg travelling with a velocity of 30 m/s.
3. A car of mass 500 kg is moving at 24 m/s. A lion of mass 100 kg drops on to the roof of the car from an overhanging branch. Show that the car will slow down to 20 m/s.
4. A car of mass 600 kg is moving at a speed of 20 m/s. It collides with a stationary car of mass 900 kg. If the first car bounces back at 4 m/s, at what speed does the second car move after the collision?
5. A ball of mass 4 kg falls to the floor; it lands with a speed of 6 m/s. It bounces off with the same speed. Show that its momentum has changed by 48 kg m/s.

3.6 Collisions

By the end of this section you should be able to:

- Distinguish between elastic and inelastic collisions.

Elastic and **inelastic** collisions will be covered in more detail in Unit 4. This short section serves as a brief introduction.

Whenever objects collide the linear momentum of the system must be conserved as long as there are no external forces acting. However, other quantities, such as **kinetic energy**, may change.

In a perfectly elastic collision the kinetic energy of the system before the collision must equal the kinetic energy of the system after the collision.

- **In an elastic collision the kinetic energy must be conserved.**

Perfectly elastic collisions are very rare. Snooker balls come pretty close but there is always a small drop in kinetic energy (most of this energy is transformed into heat and sound as the balls knock together).

A collision where the kinetic energy of the system drops after the collision is referred to as *inelastic*. Think of a tennis ball dropped on to the desk. It will bounce but it does not return to its original height as some of the kinetic energy has been lost.

Most collisions are inelastic but some are much more inelastic than others.

KEY WORDS

elastic collision *collision between two objects where the total kinetic energy is conserved*

inelastic collision *collision between two objects where the total kinetic energy is less after the collision*

kinetic energy *the energy possessed by an object as a result of its motion*

Figure 3.91 *Snooker balls produce near-perfect elastic collisions.*

Summary

In this section you have learnt that:

- Collisions can be classed as elastic or inelastic.
- In an elastic collision the kinetic of energy of the system does not change.

Review questions

1. Explain the difference between elastic and inelastic collisions.

3.7 The first condition of equilibrium

By the end of this section you should be able to:

- State the conditions required for linear equilibrium.
- Decide whether a system is in equilibrium.
- Apply the first condition of equilibrium to solve problems.

What is linear equilibrium?

Equilibrium was discussed briefly in Unit 1. In terms of forces, the first condition of linear equilibrium is when a body at rest or moving with uniform velocity has zero acceleration.

From Newton's first law, for this condition to be satisfied then the sum of all forces acting on it must be zero. In other words, there is no resultant force acting on the object.

Using the mathematical symbol ΣF for the sum of all forces we can write:

- **For linear equilibrium $\Sigma F = 0$**

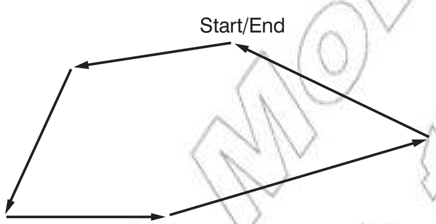


Figure 3.92 *Scale diagram showing no resultant force*

You must be careful when considering equilibrium. Free body diagrams often help here. Ensure that you have included all the forces acting on the object; don't forget weight and the contact forces acting on it.

If you draw a free body diagram and you end up back at the start then you can conclude there is no resultant force and the system is in equilibrium (remember if there are just three forces acting then they must form a triangle).

Worked example

Three forces are acting on a hovering helicopter. Its weight acts vertically downward and there is a strong horizontal wind. In order to hover, the force from the rotors must be directed slightly forward. Determine the magnitude of this force and its angle to the horizontal.

The helicopter is in equilibrium, therefore there is no net force acting on it. The three forces form a triangle, as shown in Figure 3.94.

To calculate the magnitude of the force we use Pythagoras's theorem:

$a^2 = b^2 + c^2$ *State principle or equation to be used (Pythagoras's theorem)*

$a^2 = (15\,000\text{ N})^2 + (3000\text{ N})^2$ *Substitute in known values and complete calculation*

$a = 15\,300\text{ N}$ *Clearly state the answer with unit*

To determine angle θ we use trigonometry

$\tan\theta = \text{opp} / \text{adj}$ *State principle or equation to be used (trigonometry)*

$\tan\theta = 15\,000 / 3000$ *Substitute in known values and complete calculation*

$\tan\theta = 5$ *Solve for tan θ*

$\theta = \tan^{-1} 5$ *Rearrange equation to make θ the subject and solve*

$\theta = 79°$ *Clearly state the answer with unit*

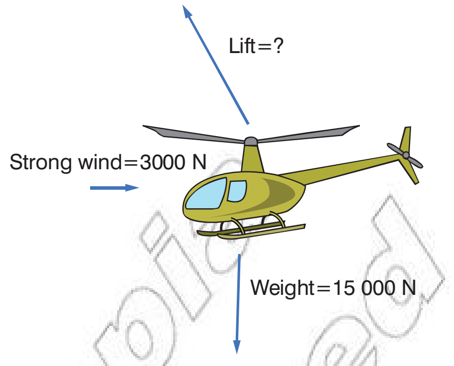


Figure 3.93 *Three forces acting on a helicopter.*

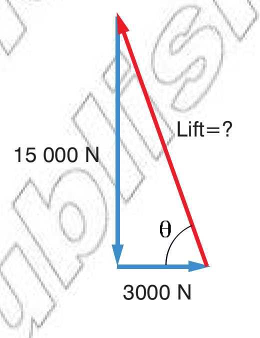


Figure 3.94 *The forces on the helicopter form a triangle.*

Summary

In this section you have learnt that:

- A system/object is in linear equilibrium if there is no resultant force acting on it.

Review questions

1. Explain what is meant by the term linear equilibrium and describe the conditions required.
2. Three forces are acting on an object in equilibrium, as shown in Figure 3.95. Either using a scale diagram or mathematically determine the magnitude and direction of force X.

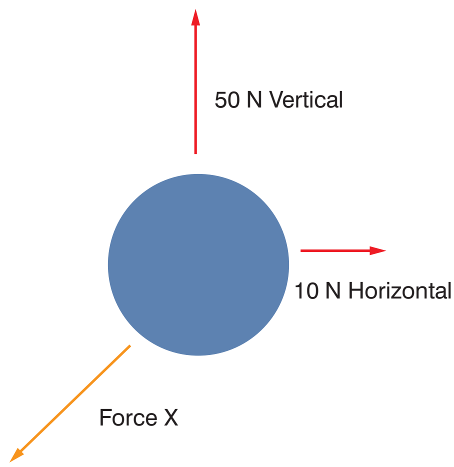


Figure 3.95 *Can you find force X?*

End of unit questions

1. State Newton's three laws of motion.
2. Explain what is meant by the term inertia and describe how it is related to mass.
3. A force of 10 N causes a spring to extend by 20 mm. Find:
   a) the spring constant of the spring in N/m
   b) the extension of the spring when 25 N is applied
   c) the force applied that causes an extension of 5 mm.
4. Calculate the weight of the following objects on Earth (assume $g = 10$ N/kg).
   a) 12 kg
   b) 500 g
   c) 20 g
   d) What is the mass and weight of each of the objects if they were placed on Mars? ($g_{Mars} = 3.8$ N/kg)
5. A runner of mass 60 kg accelerates at 2.0 m/s^2 at the start of a race. Calculate the force provided from her legs.
6. Two forces are acting on an aircraft of mass 2000 kg, as shown in Figure 3.96.

   Determine the acceleration of the aircraft.

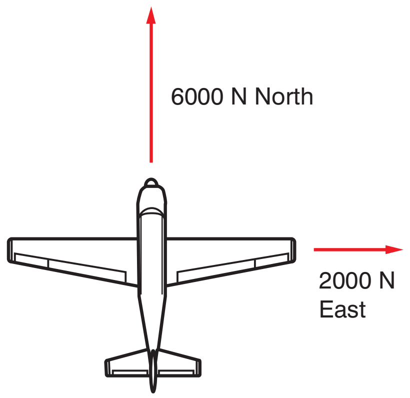


Figure 3.96 *Forces acting on an aircraft*

7. A concrete slab of mass 400 kg accelerates down a concrete slope inclined at 35°. The $\mu_{kinetic}$ between the slab and slope is 0.60. Determine the acceleration of the block.
8. State the law of conservation of linear momentum and describe its consequences.
9. A bullet of mass 0.01 kg is fired into a sandbag of mass 0.49 kg hanging from a tree. The sandbag, with the bullet embedded into it, swings away at 10 m/s. Find:
   a) the momentum after the collision
   b) the momentum before the collision
   c) the velocity of the bullet.
10. A child of mass 40 kg jumps off a wall and hits the ground at 4 m/s. He bends his knees and stops in 1 s. Calculate the force required to slow him down. How would this force be different if he didn't bend his knees and stopped in 0.1 s?